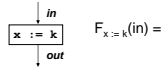
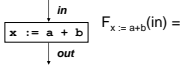


Flow functions



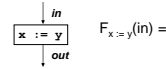
$$F_{x:=k}(in) =$$



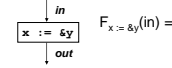
$$F_{x:=a+b}(in) =$$

7

Flow functions



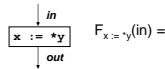
$$F_{x:=y}(in) =$$



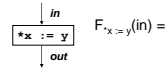
$$F_{x:=&y}(in) =$$

8

Flow functions



$$F_{x:=*y}(in) =$$



$$F_{*x:=y}(in) =$$

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Intraprocedural Points-to Analysis

- Flow functions:

$$\begin{aligned} kill(x) &= \bigcup_{v \in Vars} \{(x, v)\} \\ F_{x:=t}(S) &= S - kill(x) \\ F_{x:=a+t}(S) &= S - kill(x) \\ F_{x:=y}(S) &= S - kill(x) \cup \{(x, v) \mid (y, v) \in S\} \\ F_{x:=&y}(S) &= S - kill(x) \cup \{(x, y)\} \\ F_{x:=*y}(S) &= S - kill(x) \cup \{(x, v) \mid \exists t \in Vars. [(y, t) \in S \wedge (t, v) \in S]\} \\ F_{*x:=y}(S) &= \text{let } V := \{v \mid (x, v) \in S\} \text{ in} \\ &\quad S - (\text{if } V = \{v\} \text{ then } kill(v) \text{ else } \emptyset) \\ &\quad \cup \{(v, t) \mid v \in V \wedge (y, t) \in S\} \end{aligned}$$

10

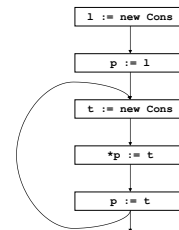
Pointers to dynamically-allocated memory

- Handle statements of the form: $x := \text{new } T$
- One idea: generate a new variable each time the new statement is analyzed to stand for the new location:

$$F_{x:=\text{new } T}(S) = S - kill(x) \cup \{(x, \text{newvar}())\}$$

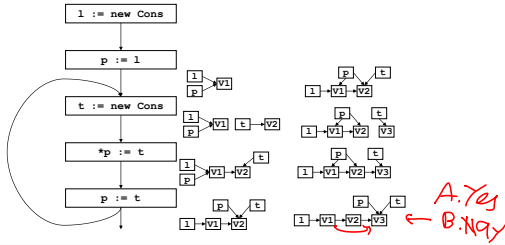
11

Example



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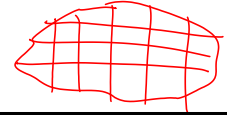
Example solved



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What went wrong?

- Lattice infinitely tall!
- We were essentially running the program
- Instead, we need to summarize the infinitely many allocated objects in a finite way
- **New Idea:** introduce summary nodes, which will stand for an entire set of allocated objects.



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What went wrong?

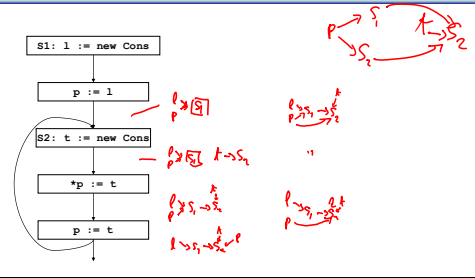
- Example: For each new statement with label L_i introduce a summary node loc_{L_i} which stands for the memory allocated by statement L_i .

$$F_{L_i}: x := new T(S) = S - kill(x) \cup \{(x, loc_{L_i})\}$$

- Summary nodes can use other criterion for merging.

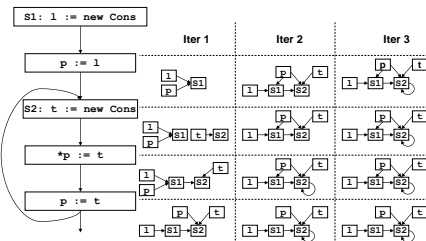
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Example revisited



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Example revisited & solved



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Array aliasing, and pointers to arrays

- Array indexing can cause aliasing:
 - $a[i]$ aliases $b[j]$ if:
 - a aliases b and $i=j$
 - a and b overlap, and $i=j+k$, where k is the amount of overlap.
- Can have pointers to elements of an array
 - $p := \&a[i]; \dots; p++;$
- How can arrays be modeled?
 - Could treat the whole array as one location.
 - Could try to reason about the array index expressions: array dependence analysis.

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Fields

- Can summarize fields using per field summary
 - for each field F, keep a points-to node called F that summarizes all possible values that can ever be stored in F
- Can also use allocation sites
 - for each field F, and each allocation site S, keep a points-to node called (F, S) that summarizes all possible values that can ever be stored in the field F of objects allocated at site S.

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Summary

- We just saw:
 - intraprocedural points-to analysis
 - handling dynamically allocated memory
 - handling pointers to arrays
- But, intraprocedural pointer analysis is not enough.
 - Sharing data structures across multiple procedures is one the big benefits of pointers: instead of passing the whole data structures around, just pass pointers to them (eg C pass by reference).
 - So pointers end up pointing to structures shared across procedures.
 - If you don't do an interproc analysis, you'll have to make conservative assumptions functions entries and function calls.

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Conservative approximation on entry

- Say we don't have interprocedural pointer analysis.
- What should the information be at the input of the following procedure:

```

global g;
void p(x,y) {
    ...
}
    
```

x

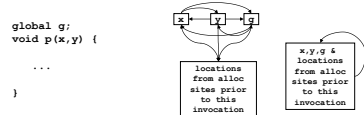
y

g

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Conservative approximation on entry

- Here are a few solutions:



- They are all very conservative!
- We can try to do better.

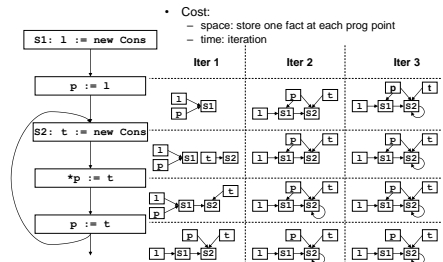
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Interprocedural pointer analysis

- Main difficulty in performing interprocedural pointer analysis is scaling
- A single points-to-graph can be $O(\text{size of program})$

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Example revisited



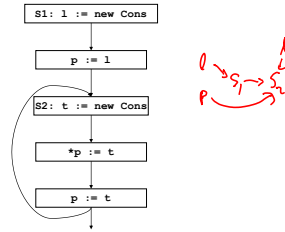
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New idea: store one dataflow fact

- Store one dataflow fact for the whole program
- Each statement updates this one dataflow fact
 - use the previous flow functions, but now they take the whole program dataflow fact, and return an updated version of it.
- Process each statement once, ignoring the order of the statements
- This is called a flow-insensitive analysis.

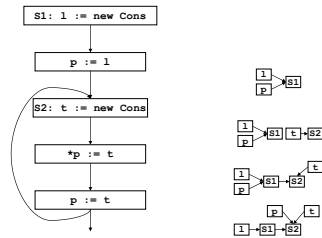
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Flow insensitive pointer analysis



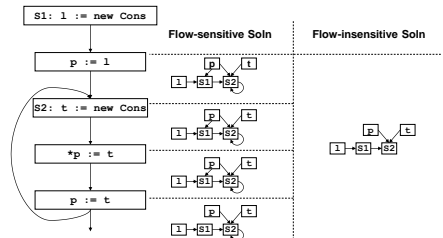
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Flow insensitive pointer analysis



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Flow sensitive vs. insensitive



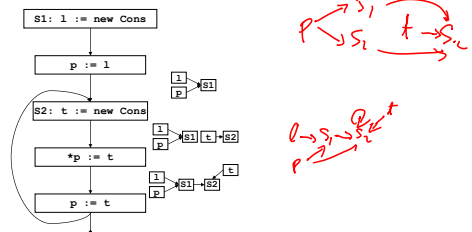
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What went wrong?

- What happened to the link between p and S1?
 - Can't do strong updates anymore!
 - Need to remove all the kill sets from the flow functions.
- What happened to the self loop on S2?
 - We still have to iterate!

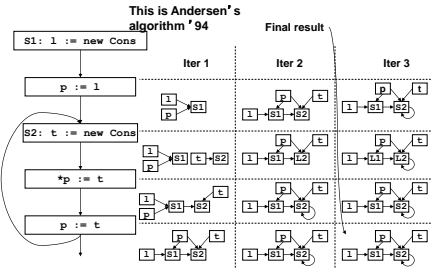
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Flow insensitive pointer analysis: fixed



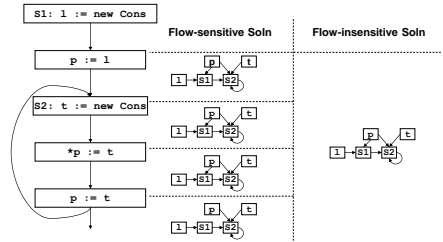
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Flow insensitive pointer analysis: fixed



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Flow sensitive vs. insensitive, again



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Flow insensitive loss of precision

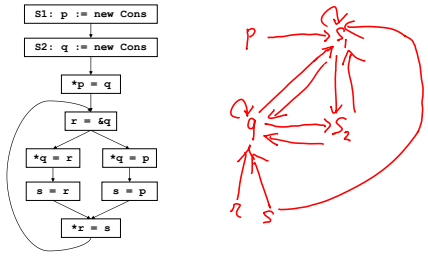
- Flow insensitive analysis leads to loss of precision!
- ```

main() {
 x := &y;
 ...
 x := &z;
}

```
- Flow insensitive analysis tells us that x may point to z here!
- However:
    - uses less memory (memory can be a big bottleneck to running on large programs)
    - runs faster

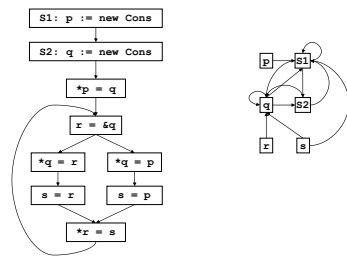
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### In Class Exercise!



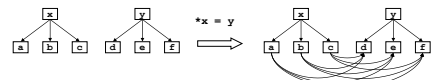
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### In Class Exercise! solved



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### Worst case complexity of Andersen

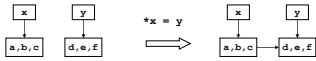


Worst case:  $N^2$  per statement, so at least  $N^3$  for the whole program. Andersen is in fact  $O(N^3)$

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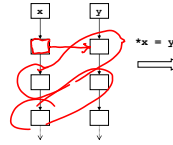
### New idea: one successor per node

- Make each node have only one successor.
- This is an invariant that we want to maintain.



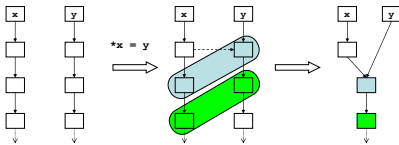
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### More general case for $*x = y$

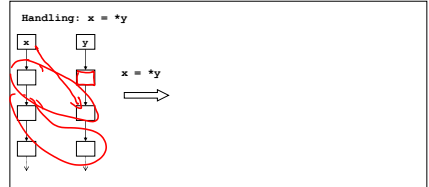


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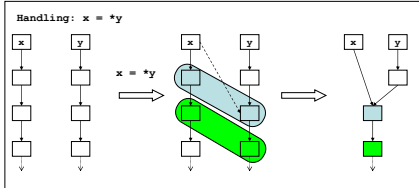
### More general case for $*x = y$



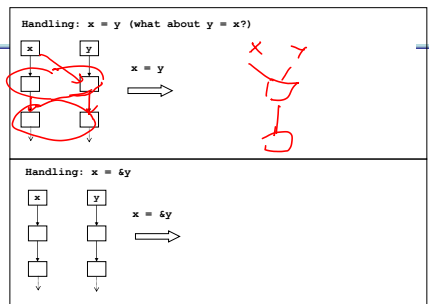
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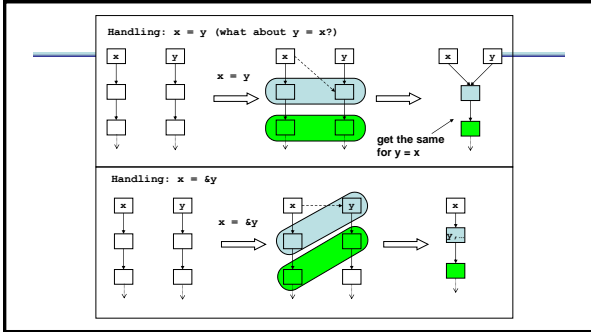
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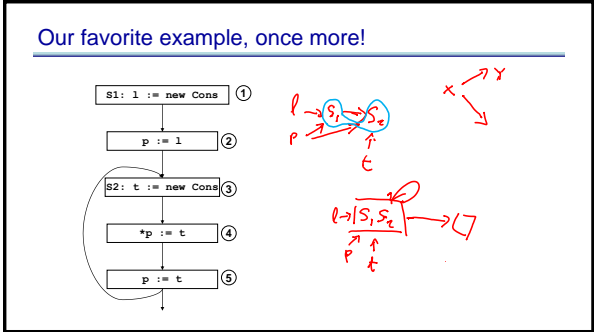
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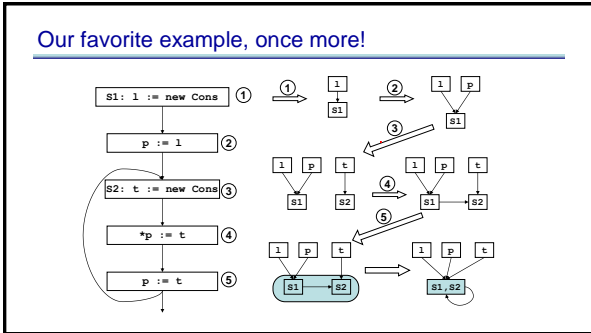
43



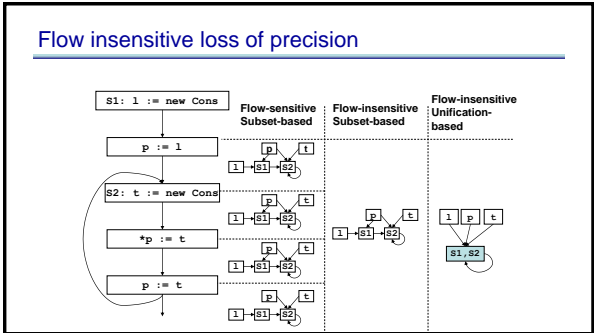
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Another example

```

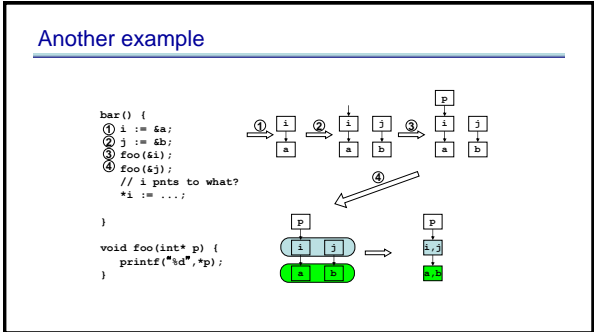
bar() {
 ① i := 5a;
 ② j := 5b;
 ③ foo(i); p:=i;
 ④ foo(j); p:=j;
 // i pnts to what?
 *i := ...;
}

void foo(int* p) {
 printf("5d", *p);
}

```

*p → i → a*

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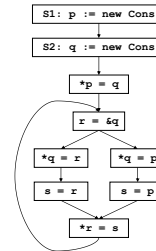


## Almost linear time

- Time complexity:  $O(N\alpha(N, N))$
- So slow-growing, it is basically linear in practice
- For the curious: node merging implemented using UNION-FIND structure, which allows set union with amortized cost of  $O(\alpha(N, N))$  per op. Take CSE 202 to learn more!

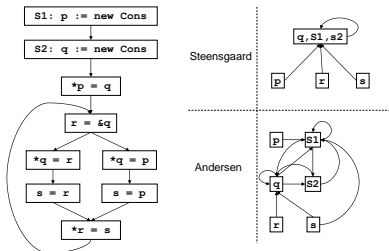
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## In Class Exercise!



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## In Class Exercise! solved



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## Advanced Pointer Analysis

- Combine flow-sensitive/flow-insensitive
- Clever data-structure design
- Context-sensitivity

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