Another example: constant prop

- Set D = $\wp(\{x \to c\})$

$\to \{a \to 2, b \to 4\}$
$a := a + b$
$\to \{a \to 6, b \to 4\}$

A. Yes
B. No
C. ??

in
`X := N`
out

$F_{X := N}(\text{in}) = \text{in} - \{x \to *\} \cup \{X \to N\}$

in
`X := Y op Z`
out

in − S_1

$\{a \to 2, b \to 4\}$
$F_{X := Y \text{ op } Z}(\text{in}) =$ (a a b) $\text{in} - \{X \to *\} \cup \{X \to C_1 \text{ op } C_2 \mid Y \to C_1 \in \text{in} \wedge Z \to C_2 \in \text{in}\}$

1

Another example: constant prop

- Set D = $2^{\{X \to N \mid X \in \text{Vars} \wedge N \in Z\}}$

in
`X := N`
out

$F_{X := N}(\text{in}) = \text{in} - \{X \to *\} \cup \{X \to N\}$

in
`X := Y op Z`
out

$F_{X := Y \text{ op } Z}(\text{in}) = \text{in} - \{X \to *\} \cup$
$\{X \to N \mid (Y \to N_1) \in \text{in} \wedge$
$(Z \to N_2) \in \text{in} \wedge$
$N = N_1 \text{ op } N_2\}$

2

Another example: constant prop

MAYPT(Y) = $\{z_1, z_2\}$

in
`X := *Y`
out

$F_{X := *Y}(\text{in}) = \text{in} - \{X \to *\} \cup \{X \to C \mid \forall Z \in \text{MAYPT}(Y).\ Z \to C \in \text{in}\}$

$\{Y \to C, Z \to C\}$

in
`*X := Y`
out

$F_{*X := Y}(\text{in}) = \text{in} - \{z \to * \mid z \in \text{MAYPT}(X)\}$

$Z \to C$

3

Another example: constant prop

in
`X := *Y`
out

$F_{X := *Y}(\text{in}) = \text{in} - \{X \to *\}$
$\cup \{X \to N \mid \forall Z \in \text{may-point-to}(Y) .$
$(Z \to N) \in \text{in}\}$

in
`*X := Y`
out

$F_{*X := Y}(\text{in}) = \text{in} - \{Z \to * \mid Z \in \text{may-point}(X)\}$
$\cup \{Z \to N \mid Z \in \text{must-point-to}(X) \wedge$
$Y \to N \in \text{in}\}$
$\cup \{Z \to N \mid (Y \to N) \in \text{in} \wedge$
$(Z \to N) \in \text{in}\}$

4

Another example: constant prop

in
`*X := *Y + *Z`
out

$F_{*X := *Y + *Z}(\text{in}) =$

in
`X := G(...)`
out

$F_{X := G(...)}(\text{in}) =$

5

Another example: constant prop

in
`*X := *Y + *Z`
out

$F_{*X := *Y + *Z}(\text{in}) = F_{a := *Y; b := *Z; c := a + b; *X := c}(\text{in})$

in
`X := G(...)`
out

$F_{X := G(...)}(\text{in}) = \emptyset$

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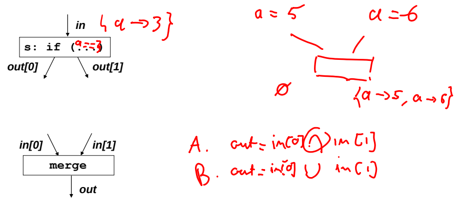
Another example: constant prop



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Lattice $\top = \emptyset$

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$

$\mathcal{P}(\{x \to c\})$

$\emptyset$ $FS = \{x \to c\}$

A $\bot = \emptyset$

B $\bot = FS$

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Lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^{A}, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \to N \mid x \in Vars \wedge N \in Z \}$

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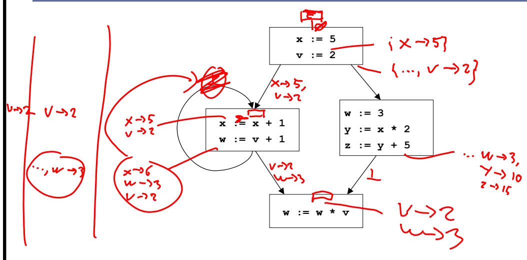
Example



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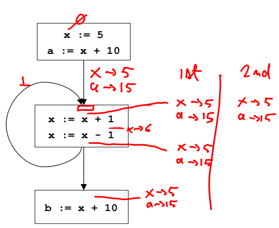
Another Example



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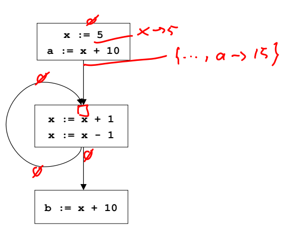
Another Example starting at top



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Back to lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^A, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \to N \mid x \in Vars \land N \in Z \}$

- What's the problem with this lattice?

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Back to lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^A, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \to N \mid x \in Vars \land N \in Z \}$

- What's the problem with this lattice?

- Lattice is infinitely high, which means we can't guarantee termination

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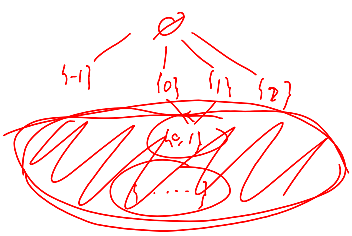
Better lattice

- Suppose we only had one variable



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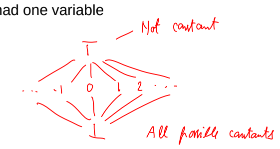
Better lattice

- Suppose we only had one variable



- $D = \{\bot, \top\} \cup Z$
- $\forall i \in Z . \bot \sqsubseteq i \land i \sqsubseteq \top$
- height = 3

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For all variables

- Two possibilities
- Option 1: Tuple of lattices
- Given lattices $(D_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1)$ ... $(D_n, \sqsubseteq_n, \bot_n, \top_n, \sqcup_n, \sqcap_n)$ create:

  tuple lattice D^n =

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For all variables

- Two possibilities
- Option 1: Tuple of lattices
- Given lattices $(D_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1)$ ... $(D_n, \sqsubseteq_n, \bot_n, \top_n, \sqcup_n, \sqcap_n)$ create:

  tuple lattice $D^n = ((D_1 \times ... \times D_n), \sqsubseteq, \bot, \top, \sqcup, \sqcap)$ where
  $\bot = (\bot_1, ..., \bot_n)$
  $\top = (\top_1, ..., \top_n)$
  $(a_1, ..., a_n) \sqcup (b_1, ..., b_n) = (a_1 \sqcup_1 b_1, ..., a_n \sqcup_n b_n)$
  $(a_1, ..., a_n) \sqcap (b_1, ..., b_n) = (a_1 \sqcap_1 b_1, ..., a_n \sqcap_n b_n)$
  height = height(D_1) + ... + height(D_n)

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For all variables

- Option 2: Map from variables to single lattice
- Given lattice (D, $\sqsubseteq_1$, $\bot_1$, $\top_1$, $\sqcup_1$, $\sqcap_1$) and a set V, create:

map lattice $V \to D = (V \to D, \sqsubseteq, \bot, \top, \sqcup, \sqcap)$

$\bot = \lambda v \to \bot_1$

$\top = \lambda v \to \top_1$

$m_1 \sqcup m_2 = \lambda v \to m_1(v) \sqcup_1 m_2(v)$

$m_1 \sqsubseteq m_2 \iff \forall v.\ m_1(v) \sqsubseteq_1 m_2(v)$

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Back to example

in

`X := Y op Z`

out

$F_{X := Y\ op\ Z}(in) =$

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Back to example

in

`X := Y op Z`

out

$F_{X := Y\ op\ Z}(in) = in\ [\ X \to in(Y)\ \widehat{op}\ in(Z)\ ]$

where a $\widehat{op}$ b =

$\widehat{op}$	$\bot$	d_1	$\top$
$\bot$	$\bot$	$\bot$	$\top$
d_2	$\bot$	$d_1\ op\ d_2$	$\top$
$\top$	$\top$	$\top$	$\top$

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General approach to domain design

- Simple lattices:
  - boolean logic lattice
  - powerset lattice
  - incomparable set: set of incomparable values, plus top and bottom (eg const prop lattice)
  - two point lattice: just top and bottom
- Use combinators to create more complicated lattices
  - tuple lattice constructor
  - map lattice constructor

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May vs Must

- Has to do with definition of computed info
- Set of x $\to$ y must-point-to pairs
  - if we compute x $\to$ y, then, then during program execution, x must point to y
- Set of x$\to$ y may-point-to pairs
  - if during program execution, it is possible for x to point to y, then we must compute x $\to$ y

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May vs must

A. $\cup$
B. $\cap$

	May	Must
most optimistic (bottom)	$\emptyset$	FS
most conservative (top)	FS	$\emptyset$
safe		
merge		

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May vs must

	May	Must
most optimistic (bottom)	empty set	full set
most conservative (top)	full set	empty set
safe	overly big	overly small
merge	∪	∩

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Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain:

$D = \mathcal{P}(\{x \to E\})$
$\top = \emptyset$
$\bot = FS$
$\sqsubseteq\ = \supseteq$
$\sqcup\ = \cap$
$\to \sqcap\ = \cup$

$a := b + c$
$-\{a \to b+c\}$
$\dots\ \cancel{b+c}\ a \dots$

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Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain:

$S = \{X \to E \mid X \in Var, E \in Expr\}$

$D := 2^S$
$\bot = S$
$\top = \emptyset$
$\sqcup = \cap$

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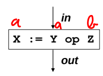
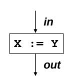
Flow functions

$a := 0$
$b := 1$
$a := a + b$
$2 \to d := \cancel{a+b}\ a$

$\emptyset\ b := 1$
$a := a + b$
$\{a \to a + b\}$



$F_{X := Y\ op\ Z}(in) = in - \{X \to *\} - \{* \to \dots X \dots\}$
$\cup \{X \to Y\ op\ Z\}$
$X := Y \wedge X\ != Z$



$F_{X := Y}(in) = in - \{X \to *\} - \{* \to \dots X \dots\}$
$\cup \{X \to E \mid Y \to E \in in\}$

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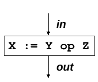
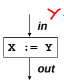
Flow functions

$(a \to b+1),$
$b := a$
$b \to b+1$



$F_{X := Y\ op\ Z}(in) = in - \{ X \to * \}$
$- \{ * \to \dots X \dots \} \cup$
$\{ X \to Y\ op\ Z \mid X \neq Y \wedge X \neq Z \}$

$Y \to X+1$



$F_{X := Y}(in) = in - \{ X \to * \}$
$- \{ * \to \dots X \dots \} \cup$
$\{ X \to E \mid Y \to E \in in \}$

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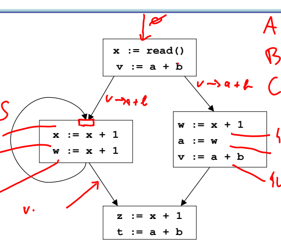
Example

A. ∪
B. ∩
C. ?



$\emptyset$

$v \to a+b$

$v \to a+b$

FS

$v \to a+b$

$v \to a+b$

$\{\dots\ w \to x+1\}$

v.

$\{\dots, w \to x+1\}$

$\{a \to x+1, w \to x+1\}$

$\{v \to a+b, a \to x+1, w \to x+1\}$

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Direction of analysis

- Although constraints are not directional, flow functions are
- All flow functions we have seen so far are in the forward direction
- In some cases, the constraints are of the form
  in = F(out)
- These are called backward problems.
- Example: live variables
  – compute the set of variables that may be live

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Live Variables

- A variable is live at a program point if it will be used before being redefined
- A variable is dead at a program point if it is redefined before being used

a = 1
A. b = 1
B. print (a, b)
C. b = 10
D a = 20
E print (a, b)

a = 100
...
print (a)

print (a)

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Example: live variables

- Set D =
- Lattice: $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap)$ =

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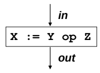
Example: live variables

- Set D = 2^{Vars}
- Lattice: $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) = (2^{\text{Vars}}, \subseteq, \emptyset, \text{Vars}, \cup, \cap)$



$F_{X := Y \text{ op } Z}(\text{out})$ =

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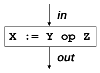
Example: live variables

- Set D = 2^{Vars}
- Lattice: $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) = (2^{\text{Vars}}, \subseteq, \emptyset, \text{Vars}, \cup, \cap)$



$F_{X := Y \text{ op } Z}(\text{out}) = \text{out} - \{ X \} \cup \{ Y, Z \}$

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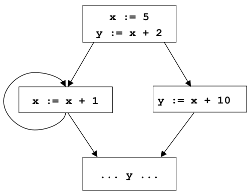
Example: live variables



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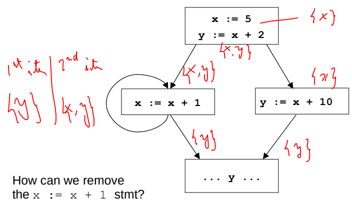
Example: live variables



How can we remove the `x := x + 1` stmt?

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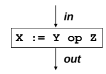
Revisiting assignment



$F_{X := Y \text{ op } Z}(\text{out}) = \text{out} - \{ X \} \cup \{ Y, Z \}$

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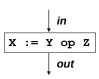
Revisiting assignment



$F_{X := Y \text{ op } Z}(\text{out}) = \text{out} - \{ X \} \cup \{ Y, Z \}$

$\text{out} - \{x\} \cup$

$x \notin \text{out} \,?\, \emptyset : \{Y, Z\}$

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Theory of backward analyses

- Can formalize backward analyses in two ways
- Option 1: reverse flow graph, and then run forward problem
- Option 2: re-develop the theory, but in the backward direction

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Precision

- Going back to constant prop, in what cases would we lose precision?

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Precision

- Going back to constant prop, in what cases would we lose precision?

```
x := 5
if (<expr>) {
    x := 6
}
... x ...

where <expr> is
equiv to false
```

```
if (p) {
    x := 5;
} else
    x := 4;
}
...
if (p) {
    y := x + 1
} else {
    y := x + 2
}
... y ...
```

```
if (...) {
    x := -1;
} else
    x := 1;
}
y := x * x;
... y ...
```

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Precision

- The first problem: Unreachable code
  - solution: run unreachable code removal before
  - the unreachable code removal analysis will do its best, but may not remove all unreachable code
- The other two problems are path-sensitivity issues
  - Branch correlations: some paths are infeasible
  - Path merging: can lead to loss of precision

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MOP: meet over all paths

- Information computed at a given point is the meet of the information computed by each path to the program point

```
if (...) {
   x := -1;
} else
   x := 1;
}
y := x * x;
... y ...
```

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MOP

- For a path p, which is a sequence of statements $[s_1, ..., s_n]$, define: $F_p(in) = F_{s_n}(...F_{s_1}(in) ...)$
- In other words: $F_p =$ $F_{s_1} \circ \cdots \circ F_{s_n}$
- Given an edge e, let paths-to(e) be the (possibly infinite) set of paths that lead to e
- Given an edge e, MOP(e) = $\bigsqcup_{p \in paths\text{-}to(e)} F_p(\bot)$
- For us, should be called JOP (ie: join, not meet)

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MOP vs. dataflow

- MOP is the "best" possible answer, given a fixed set of flow functions
  - This means that MOP $\sqsubseteq$ dataflow at edge in the CFG
- In general, MOP is not computable (because there can be infinitely many paths)
  - vs dataflow which is generally computable (if flow fns are monotonic and height of lattice is finite)
- And we saw in our example, in general, MOP $\neq$ dataflow

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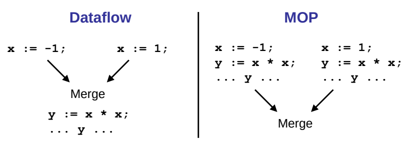
MOP vs. dataflow

- However, it would be great if by imposing some restrictions on the flow functions, we could guarantee that dataflow is the same as MOP. What would this restriction be?



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MOP vs. dataflow

- However, it would be great if by imposing some restrictions on the flow functions, we could guarantee that dataflow is the same as MOP. What would this restriction be?
- Distributive problems. A problem is distributive if:

  $\forall$ a, b . $F(a \sqcup b) = F(a) \sqcup F(b)$

- If flow function is distributive, then MOP = dataflow

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Summary of precision

- Dataflow is the basic algorithm
- To basic dataflow, we can add path-separation
  - Get MOP, which is same as dataflow for distributive problems
  - Variety of research efforts to get closer to MOP for non-distributive problems
- To basic dataflow, we can add path-pruning
  - Get branch correlation
- To basic dataflow, can add both:
  - meet over all feasible paths

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