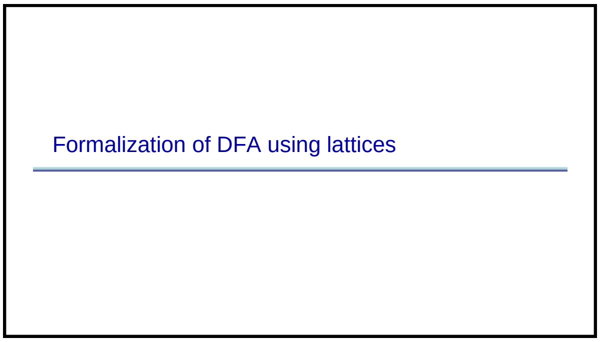
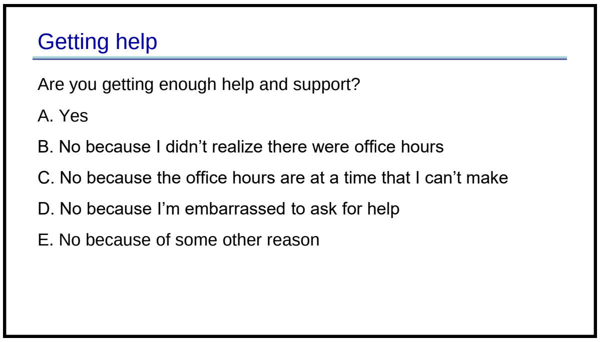


Formalization of DFA using lattices

1



Getting help

Are you getting enough help and support?

A. Yes

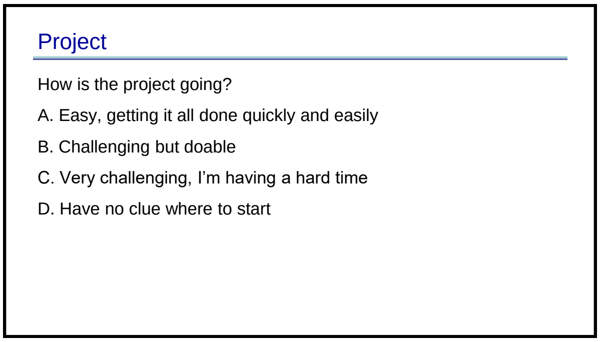
B. No because I didn’t realize there were office hours

C. No because the office hours are at a time that I can’t make

D. No because I’m embarrassed to ask for help

E. No because of some other reason

2



Project

How is the project going?

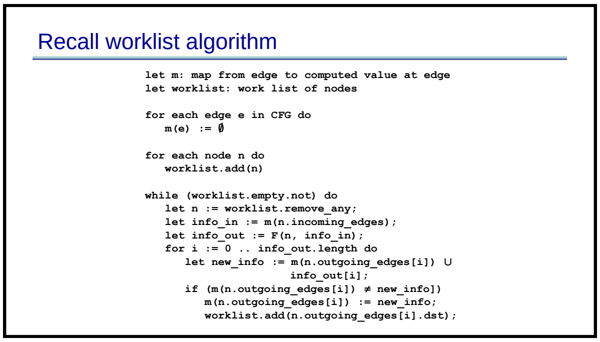
A. Easy, getting it all done quickly and easily

B. Challenging but doable

C. Very challenging, I’m having a hard time

D. Have no clue where to start

3



Recall worklist algorithm

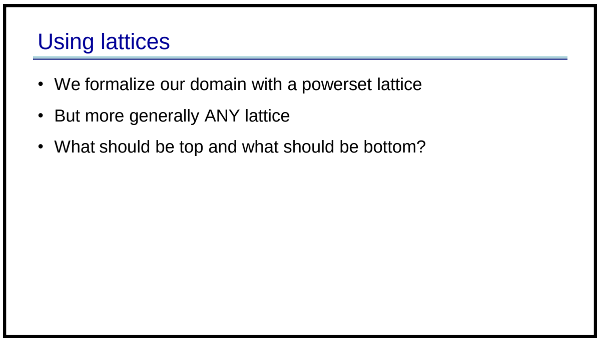
```
let m: map from edge to computed value at edge
let worklist: work list of nodes

for each edge e in CFG do
   m(e) := ∅

for each node n do
   worklist.add(n)

while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length do
      let new_info := m(n.outgoing_edges[i]) ∪
                      info_out[i];
      if (m(n.outgoing_edges[i]) ≠ new_info])
         m(n.outgoing_edges[i]) := new_info;
         worklist.add(n.outgoing_edges[i].dst);
```

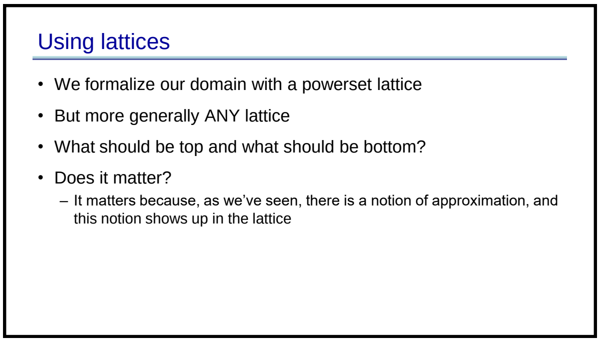
4



Using lattices

- We formalize our domain with a powerset lattice
- But more generally ANY lattice
- What should be top and what should be bottom?

5



Using lattices

- We formalize our domain with a powerset lattice
- But more generally ANY lattice
- What should be top and what should be bottom?
- Does it matter?
  - It matters because, as we’ve seen, there is a notion of approximation, and this notion shows up in the lattice

6

Using lattices

- Unfortunately:
  - – dataflow analysis community has picked one direction
  - – abstract interpretation community has picked the other
- We will work with the abstract interpretation direction
- Bottom is the most precise (optimistic) answer, Top the most imprecise (conservative)

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Direction of lattice

- Always safe to go up in the lattice
- Can always set the result to ⊤
- Hard to go down in the lattice
- Bottom will be the empty set in reaching defs

8

Worklist algorithm using lattices

```
let m: map from edge to computed value at edge
let worklist: work list of nodes

for each edge e in CFG do
   m(e) := ⊥

for each node n do
   worklist.add(n)

while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length do
      let new_info := m(n.outgoing_edges[i]) ⊔
                      info_out[i];
      if (m(n.outgoing_edges[i]) ≠ new_info])
         m(n.outgoing_edges[i]) := new_info;
         worklist.add(n.outgoing_edges[i].dst);
```

9

Termination of this algorithm?

- For reaching definitions, it terminates...
- Why?
  - – lattice is finite
- Can we loosen this requirement?

10

Termination of this algorithm?

- For reaching definitions, it terminates...
- Why?
  - – lattice is finite
- Can we loosen this requirement?
  - – Yes, we only require the lattice to have a finite height
- Height of a lattice: length of the longest ascending or descending chain
- Height of lattice (2^S, ⊆) = ??

A. | S | - 1
B. | S |
C. | S | + 1
D. None of the above

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Termination of this algorithm?

- For reaching definitions, it terminates...
- Why?
  - – lattice is finite
- Can we loosen this requirement?
  - – Yes, we only require the lattice to have a finite height
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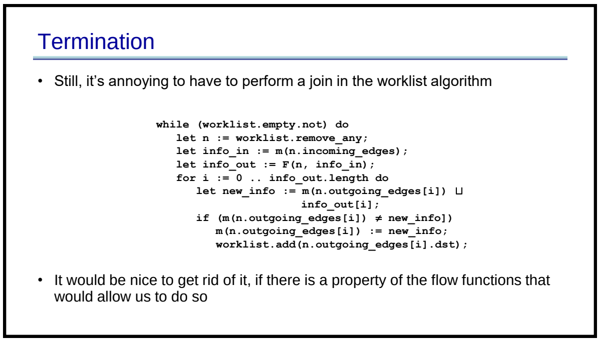
12



Termination

- Still, it’s annoying to have to perform a join in the worklist algorithm

```
while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length do
      let new_info := m(n.outgoing_edges[i]) ⊔
                      info_out[i];
      if (m(n.outgoing_edges[i]) ≠ new_info])
         m(n.outgoing_edges[i]) := new_info;
         worklist.add(n.outgoing_edges[i].dst);
```

- It would be nice to get rid of it, if there is a property of the flow functions that would allow us to do so

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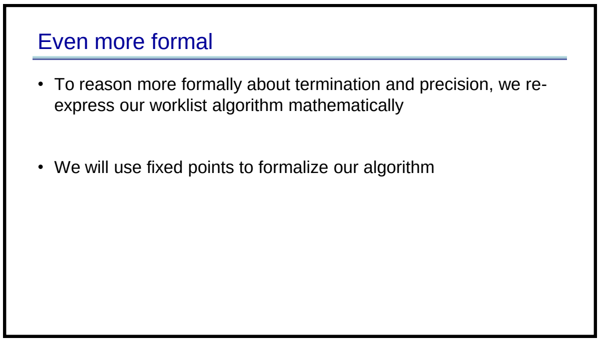

Even more formal

- To reason more formally about termination and precision, we re-express our worklist algorithm mathematically
- We will use fixed points to formalize our algorithm

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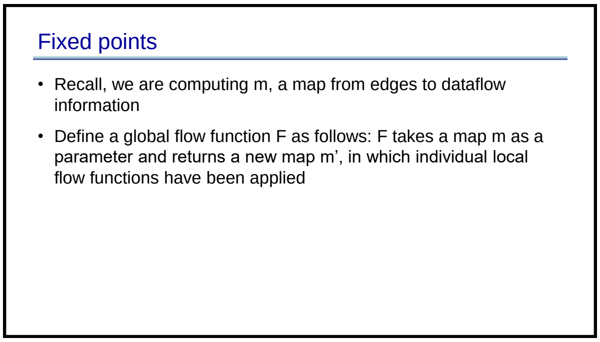

Fixed points

- Recall, we are computing m, a map from edges to dataflow information
- Define a global flow function F as follows: F takes a map m as a parameter and returns a new map m’, in which individual local flow functions have been applied

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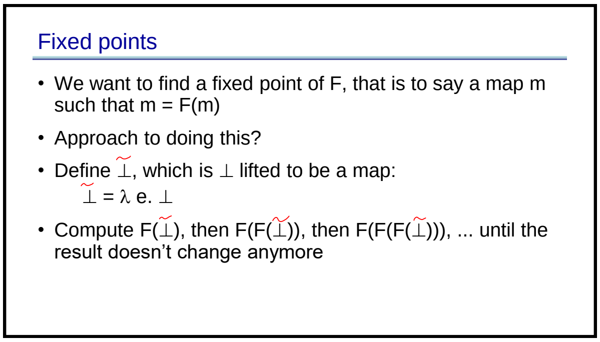

Fixed points

- We want to find a fixed point of F, that is to say a map m such that m = F(m)
- Approach to doing this?
- Define $\tilde{\bot}$, which is ⊥ lifted to be a map:
  $\tilde{\bot} = \lambda$ e. ⊥
- Compute F($\tilde{\bot}$), then F(F($\tilde{\bot}$)), then F(F(F($\tilde{\bot}$))), ... until the result doesn’t change anymore

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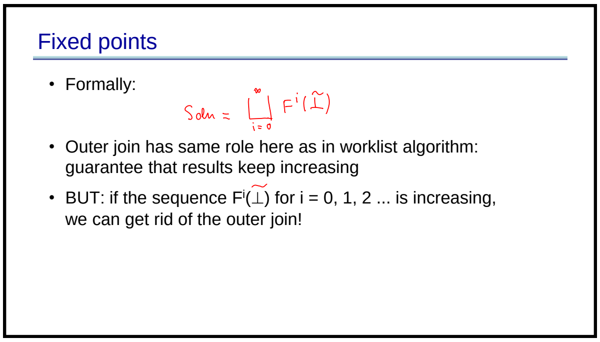

Fixed points

- Formally:

$$\text{Soln} = \bigsqcup_{i=0}^{\infty} F^i(\tilde{\bot})$$

- Outer join has same role here as in worklist algorithm: guarantee that results keep increasing
- BUT: if the sequence $F^i(\tilde{\bot})$ for i = 0, 1, 2 ... is increasing, we can get rid of the outer join!

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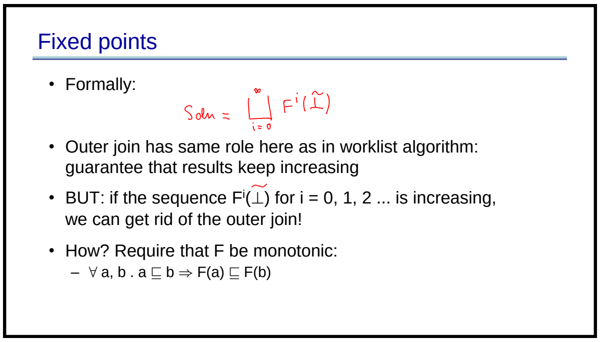

Fixed points

- Formally:

$$\text{Soln} = \bigsqcup_{i=0}^{\infty} F^i(\tilde{\bot})$$

- Outer join has same role here as in worklist algorithm: guarantee that results keep increasing
- BUT: if the sequence $F^i(\tilde{\bot})$ for i = 0, 1, 2 ... is increasing, we can get rid of the outer join!
- How? Require that F be monotonic:
  - $\forall$ a, b . a $\sqsubseteq$ b $\Rightarrow$ F(a) $\sqsubseteq$ F(b)

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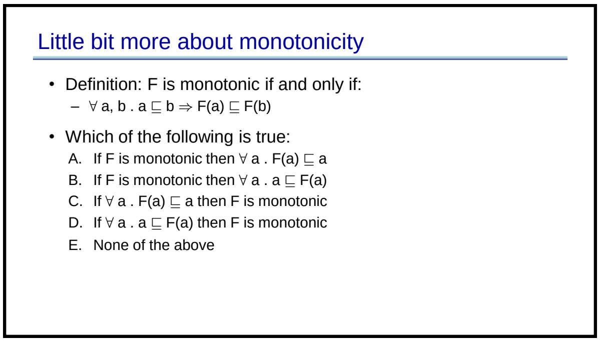

Little bit more about monotonicity

- Definition: F is monotonic if and only if:
  - ∀ a, b . a ⊑ b ⇒ F(a) ⊑ F(b)
- Which of the following is true:
  - A. If F is monotonic then ∀ a . F(a) ⊑ a
  - B. If F is monotonic then ∀ a . a ⊑ F(a)
  - C. If ∀ a . F(a) ⊑ a then F is monotonic
  - D. If ∀ a . a ⊑ F(a) then F is monotonic
  - E. None of the above

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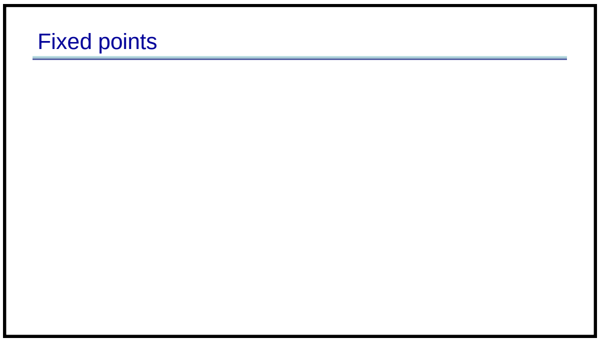

Fixed points

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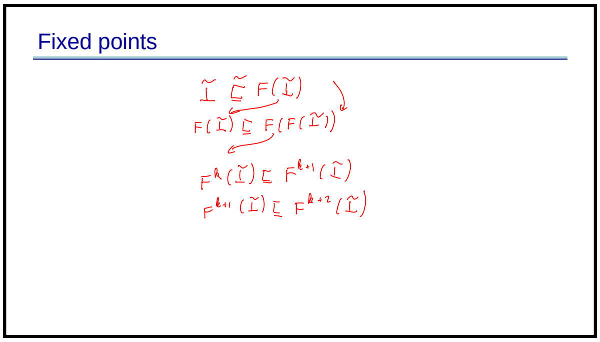

Fixed points

$$\tilde{I} \,\tilde{\sqsubseteq}\, F(\tilde{I})$$

$$F(\tilde{I}) \sqsubseteq F(F(\tilde{I}))$$

$$F^{k}(\tilde{I}) \sqsubseteq F^{k+1}(\tilde{I})$$

$$F^{k+1}(\tilde{I}) \sqsubseteq F^{k+2}(\tilde{I})$$

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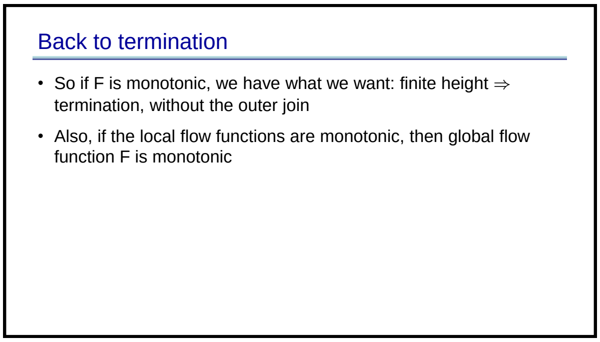

Back to termination

- So if F is monotonic, we have what we want: finite height ⇒ termination, without the outer join
- Also, if the local flow functions are monotonic, then global flow function F is monotonic

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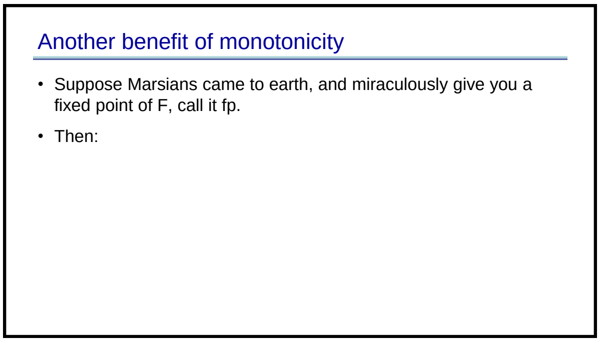

Another benefit of monotonicity

- Suppose Marsians came to earth, and miraculously give you a fixed point of F, call it fp.
- Then:

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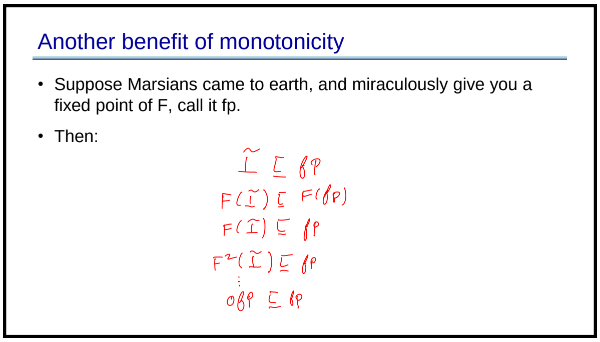

Another benefit of monotonicity

- Suppose Marsians came to earth, and miraculously give you a fixed point of F, call it fp.
- Then:

$$\tilde{I} \sqsubseteq fp$$

$$F(\tilde{I}) \sqsubseteq F(fp)$$

$$F(\tilde{I}) \sqsubseteq fp$$

$$F^{2}(\tilde{I}) \sqsubseteq fp$$

$$\vdots$$

$$ofp \sqsubseteq fp$$

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Another benefit of monotonicity

- We are computing the least fixed point...

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Recap

- Let’s do a recap of what we’ve seen so far

- Started with worklist algorithm for reaching definitions

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Worklist algorithm for reaching defns

```
let m: map from edge to computed value at edge
let worklist: work list of nodes

for each edge e in CFG do
   m(e) := ∅

for each node n do
   worklist.add(n)

while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length do
      let new_info := m(n.outgoing_edges[i]) ∪
                      info_out[i];
      if (m(n.outgoing_edges[i]) ≠ new_info])
         m(n.outgoing_edges[i]) := new_info;
         worklist.add(n.outgoing_edges[i].dst);
```

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Generalized algorithm using lattices

```
let m: map from edge to computed value at edge
let worklist: work list of nodes

for each edge e in CFG do
   m(e) := ⊥

for each node n do
   worklist.add(n)

while (worklist.empty.not) do
   let n := worklist.remove_any;
   let info_in := m(n.incoming_edges);
   let info_out := F(n, info_in);
   for i := 0 .. info_out.length do
      let new_info := m(n.outgoing_edges[i]) ⊔
                      info_out[i];
      if (m(n.outgoing_edges[i]) ≠ new_info])
         m(n.outgoing_edges[i]) := new_info;
         worklist.add(n.outgoing_edges[i].dst);
```

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Next step: removed outer join

- Wanted to remove the outer join, while still providing termination guarantee

- To do this, we re-expressed our algorithm more formally

- We first defined a “global” flow function F, and then expressed our algorithm as a fixed point computation

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Guarantees

- If F is monotonic, don’t need outer join
- If F is monotonic and height of lattice is finite: iterative algorithm terminates
- If F is monotonic, the fixed point we find is the least fixed point.

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What about if we start at top?

- What if we start with $\tilde{\top}$: $F(\tilde{\top})$, $F(F(\tilde{\top}))$, $F(F(F(\tilde{\top})))$

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What about if we start at top?

- What if we start with $\tilde{\top}$: $F(\tilde{\top})$, $F(F(\tilde{\top}))$, $F(F(F(\tilde{\top})))$
- We get the greatest fixed point
- Why do we prefer the least fixed point?
  - More precise

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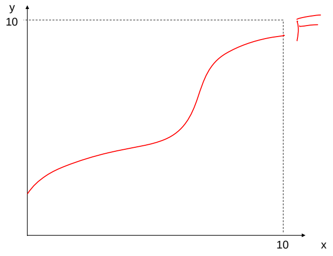
Graphically



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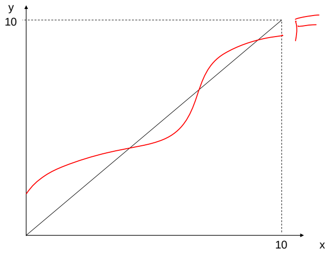
Graphically



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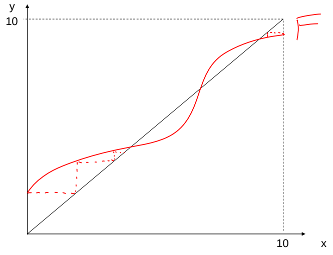
Graphically



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Graphically, another way

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