Program Representations

Representing programs

- Goals

Representing programs

- Primary goals
  - analysis is easy and effective
    - just a few cases to handle
    - directly link related things
  - transformations are easy to perform
  - general, across input languages and target machines
- Additional goals
  - compact in memory
  - easy to translate to and from
  - tracks info from source through to binary, for source-level debugging, profilling, typed binaries
  - extensible (new opts, targets, language features)
  - displayable

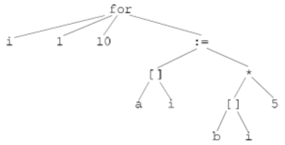
Option 1: high-level syntax based IR

- Represent source-level structures and expressions directly
- Example: Abstract Syntax Tree

Source:

```
for i := 1 to 10 do
  a[i] := b[i] * 5;
end
```

AST:



Option 2: low-level IR

- Translate input programs into low-level primitive chunks, often close to the target machine
- Examples: assembly code, virtual machine code (e.g. stack machines), three-address code, register-transfer language (RTL)
- Standard RTL instrs:

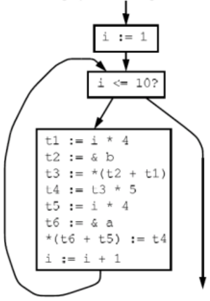
assignment	`x := y;`
unary op	`x := op y;`
binary op	`x := y op z;`
address-of	`p := &y;`
load	`x := *(p + o);`
store	`*(p + o) := x;`
call	`x := f(...);`
unary compare	`op x ?`
binary compare	`x op y ?`

Option 2: low-level IR

Source:

```
for i := 1 to 10 do
  a[i] := b[i] * 5;
end
```

Control flow graph containing RTL instructions:



Comparison

Comparison

- Advantages of high-level rep
  - analysis can exploit high-level knowledge of constructs
  - easy to map to source code (debugging, profiling)
- Advantages of low-level rep
  - can do low-level, machine specific reasoning
  - can be language-independent
- Can mix multiple reps in the same compiler

Components of representation

- Control dependencies: sequencing of operations
  - evaluation of if & then
  - side-effects of statements occur in right order
- Data dependencies: flow of definitions from defs to uses
  - operands computed before operations
- Ideal: represent just dependencies that matter
  - dependencies constrain transformations
  - fewest dependences ⇒ flexibility in implementation

X:=5
Y:=6

Control dependencies

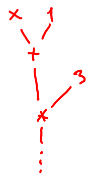
- Option 1: high-level representation
  - control implicit in semantics of AST nodes
- Option 2: control flow graph (CFG)
  - nodes are individual instructions
  - edges represent control flow between instructions
- Options 2b: CFG with basic blocks
  - basic block: sequence of instructions that don’t have any branches, and that have a single entry point
  - BB can make analysis more efficient: compute flow functions for an entire BB before start of analysis

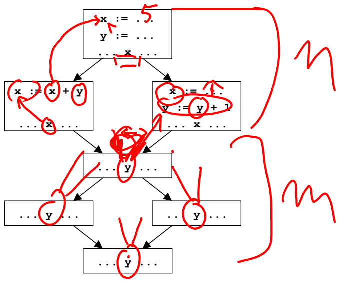
Control dependencies

- CFG does not capture loops very well
- Some fancier options include:
  - the Control Dependence Graph
  - the Program Dependence Graph
- More on this later. Let’s first look at data dependencies

Data dependencies

- Simplest way to represent data dependencies: def/use chains



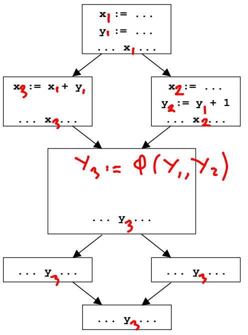


Def/use chains

- Directly captures dataflow
  - works well for things like constant prop
- But...
- Ignores control flow
  - misses some opt opportunities since conservatively considers all paths
  - not executable by itself (for example, need to keep CFG around)
  - not appropriate for code motion transformations
- Must update after each transformation
- Space consuming

SSA

- Static Single Assignment
  - invariant: each use of a variable has only one def



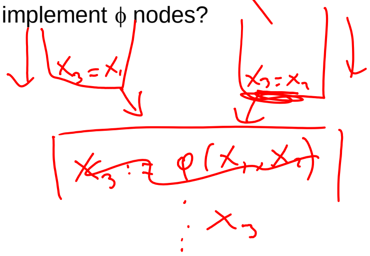
SSA

- Create a new variable for each def
- Insert ϕ pseudo-assignments at merge points
- Adjust uses to refer to appropriate new names

- Question: how can one figure out where to insert ϕ nodes using a liveness analysis and a reaching defns analysis.

Converting back from SSA

- Semantics of $x_3 := \phi(x_1, x_2)$
  - set x_3 to x_i if execution came from ith predecessor
- How to implement ϕ nodes?



Converting back from SSA

- Semantics of $x_3 := \phi(x_1, x_2)$
  - set x_3 to x_i if execution came from ith predecessor
- How to implement ϕ nodes?
  - Insert assignment $x_3 := x_1$ along 1st predecessor
  - Insert assignment $x_3 := x_2$ along 2nd predecessor
- If register allocator assigns x_1, x_2 and x_3 to the same register, these moves can be removed
  - x_1 .. x_n usually have non-overlapping lifetimes, so this kind of register assignment is legal

Recall: Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain: $\{x \to E_1, y \to E_2, z \to E_3\}$

$S = \{x \to E \mid x \in Var, E \in Expr\}$

$D = 2^S$

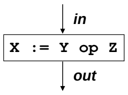
$\bot = S$

$\top = \emptyset$

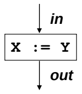
$\sqcup = \cap$

Recall: CSE Flow functions

$a_1 := a_2 + b$

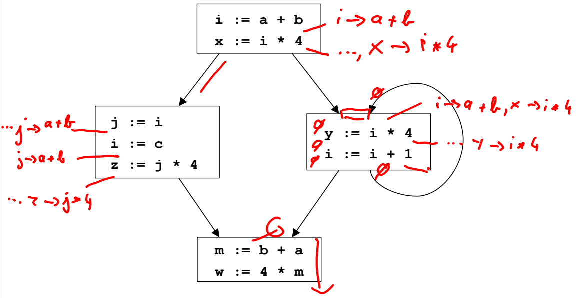


$F_{X := Y \text{ op } Z}(\text{in}) = \text{in} - \{ X \to * \} - \{ * \to \ldots X \ldots \} \cup \{ X \to Y \text{ op } Z \mid X \neq Y \wedge X \neq Z\}$

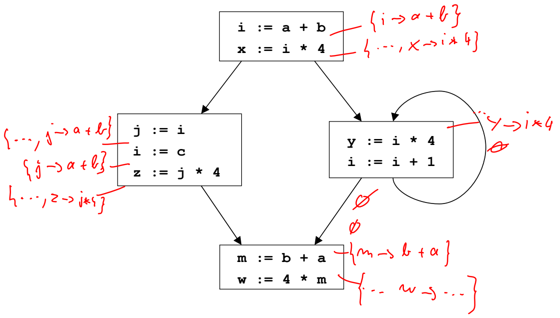


$F_{X := Y}(\text{in}) = \text{in} - \{ X \to * \} - \{ * \to \ldots X \ldots \} \cup \{ X \to E \mid Y \to E \in \text{in} \}$

Example



Example



Problems

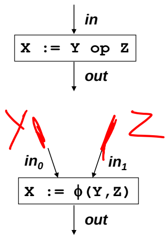
- z := j * 4 is not optimized to z := x, even though x contains the value j * 4
- m := b + a is not optimized, even though a + b was already computed
- w := 4 * m it not optimized to w := x, even though x contains the value 4 *m

---

Problems: more abstractly

- Available expressions overly sensitive to name choices, operand orderings, renamings, assignments
- Use SSA: distinct values have distinct names
- Do copy prop before running available exprs
- Adopt canonical form for commutative ops

Example in SSA

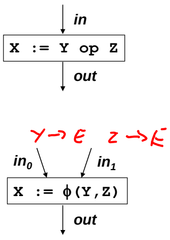


$F_{X := Y\ op\ Z}(in) =$

$F_{X := \phi(Y,Z)}(in_0, in_1) =$

$\underbrace{\{Y \to E\} \quad \{Z \to E\}}_{X \to E'}$

Example in SSA

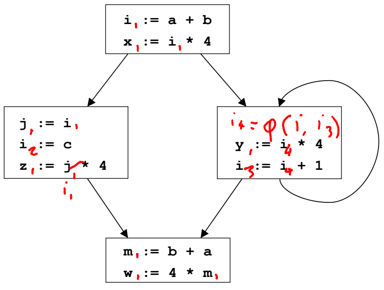


$F_{X := Y\ op\ Z}(in) = in \cup \{ X \to Y\ op\ Z \}$

$F_{X := \phi(Y,Z)}(in_0, in_1) = (in_0 \cap in_1) \cup$

$\{ X \to E \mid Y \to E \in in_0 \land Z \to E \in in_1 \}$

Example in SSA



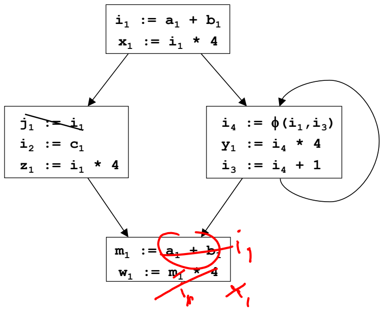
```
i1 := a + b
x1 := i1 * 4
```

```
j1 := i1
i2 := c
z1 := j1 * 4
```

```
i4 := φ(i1, i3)
y1 := i4 * 4
i3 := i4 + 1
```

```
m1 := b + a
w1 := 4 * m1
```

Example in SSA



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z1 := i1 * 4
```

```
i4 := ϕ(i1,i3)
y1 := i4 * 4
i3 := i4 + 1
```

```
m1 := a1 + b1
w1 := m1 * 4
```

What about pointers?

- Pointers complicate SSA. Several options.
- Option 1: don’t use SSA for pointed to variables
- Option 2: adapt SSA to account for pointers
- Option 3: define src language so that variables cannot be pointed to (eg: Java)

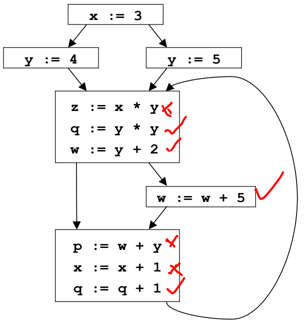
SSA helps us with CSE

- Let’s see what else SSA can help us with
- Loop-invariant code motion

Loop-invariant code motion

- Two steps: analysis and transformations
- Step1: find invariant computations in loop
  - invariant: computes same result each time evaluated
- Step 2: move them outside loop
  - to top if used within loop: **code hoisting**
  - to bottom if used after loop: **code sinking**

Example



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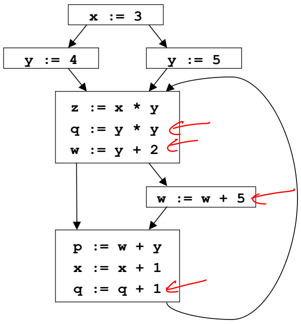


Detecting loop invariants

rand(a)

- An expression is invariant in a loop L iff:

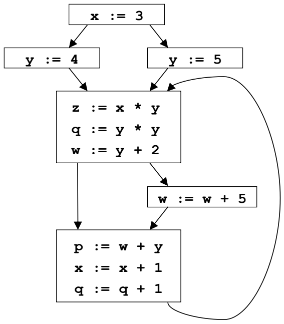
(base cases)
- it's a constant
- it's a variable use, all of whose defs are outside of L

(inductive cases)
- it's a pure computation all of whose args are loop-invariant
- it's a variable use with only one reaching def, and the rhs of that def is loop-invariant

Computing loop invariants

- Option 1: iterative dataflow analysis
  - optimistically assume all expressions loop-invariant, and propagate
- Option 2: build def/use chains
  - follow chains to identify and propagate invariant expressions
- Option 3: SSA
  - like option 2, but using SSA instead of def/use chains

Example using def/use chains



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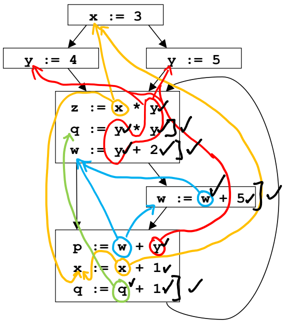

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Loop invariant detection using SSA

- An expression is invariant in a loop L iff:

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  - it's a constant
  - it's a variable use, all of whose **single** defs are outside of L

(inductive cases)

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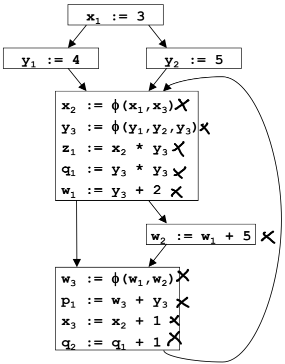
- ϕ functions are not pure

Example using SSA



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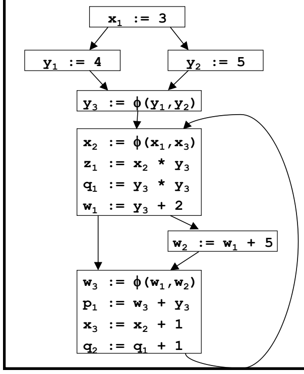
Example using SSA and preheader



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  - it's a variable use whose **single** reading def, and the rhs of that def is loop-invariant

- ϕ functions are not pure

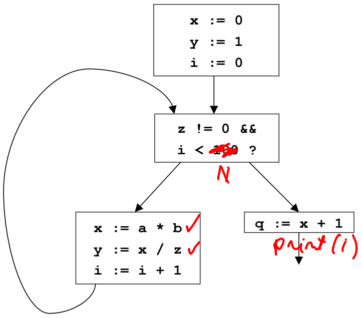
Summary: Loop-invariant code motion

- Two steps: analysis and transformations
- Step1: find invariant computations in loop
  - invariant: computes same result each time evaluated
- Step 2: move them outside loop
  - to top if used within loop: **code hoisting**
  - to bottom if used after loop: **code sinking**

Code motion

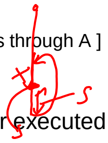
- Say we found an invariant computation, and we want to move it out of the loop (to loop pre-header)
- When is it legal?
- Need to preserve relative order of invariant computations to preserve data flow among move statements
- Need to preserve relative order between invariant computations and other computations

Example



Lesson from example: domination restriction

- To move statement S to loop pre-header, S must **dominate** all loop exits
  - [A dominates B when all paths to B first pass through A]



- Otherwise may execute S when never executed otherwise
- If S is pure, then can relax this constraint at cost of possibly slowing down the program

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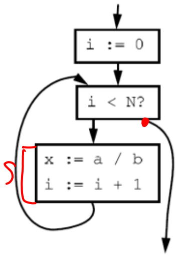
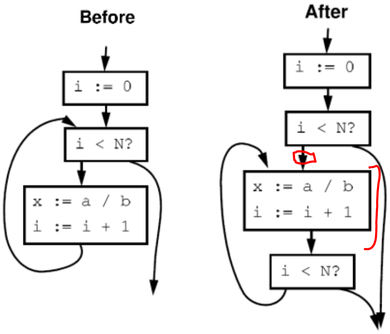
Domination restriction in for loops



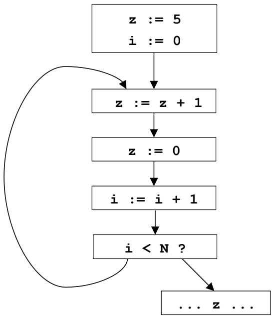
Domination restriction in for loops



Avoiding domination restriction

- Domination restriction strict
  - Nothing inside branch can be moved
  - Nothing after a loop exit can be moved
- Can be circumvented through loop normalization
  - while-do => if-do-while

Another example



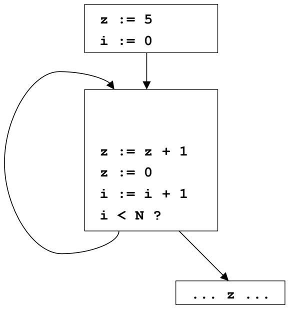
Data dependence restriction

- To move S: `z := x op y`:

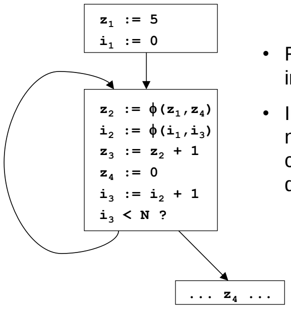
  S must be the only assignment to `z` in loop, and no use of `z` in loop reached by any def other than S

- Otherwise may reorder defs/uses

Avoiding data restriction



Avoiding data restriction



- Restriction unnecessary in SSA!!!
- Implementation of phi nodes as moves will cope with re-ordered defs/uses

---

Summary of Data dependencies

- We've seen SSA, a way to encode data dependencies better than just def/use chains
  - makes CSE easier
  - makes loop invariant detection easier
  - makes code motion easier
- Now we move on to looking at how to encode control dependencies

---

Control Dependencies

- A node (basic block) Y is control-dependent on another X iff X determines whether Y executes
  - there exists a path from X to Y s.t. every node in the path other than X and Y is post-dominated by Y
  - X is not post-dominated by Y

Control Dependencies

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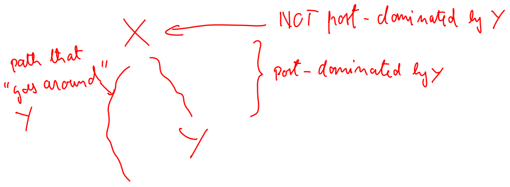
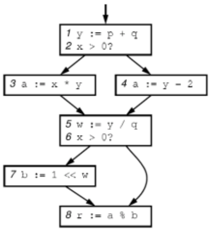
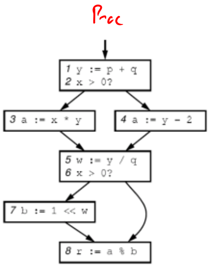


Example

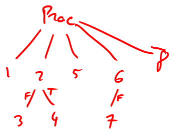


3 D 2
4 D 2
7 D 6

Example



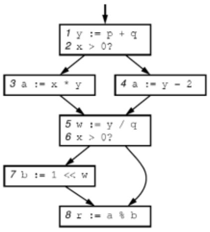
Control dependence relation
3 depends on 2
4 " " 2
7 " " 6



Control Dependence Graph

- Control dependence graph: Y descendent of X iff Y is control dependent on X
  - label each child edge with required condition
  - group all children with same condition under region node
- Program dependence graph: super-impose dataflow graph (in SSA form or not) on top of the control dependence graph

Example



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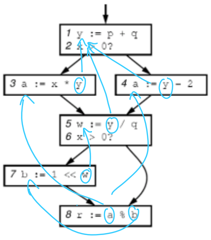

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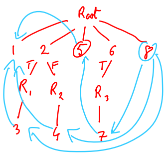
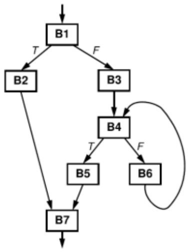


Another example



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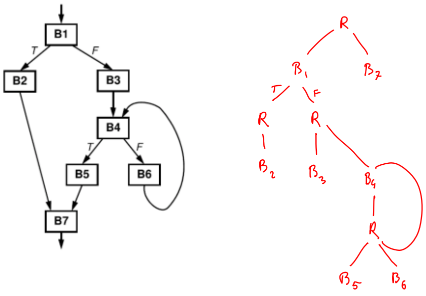
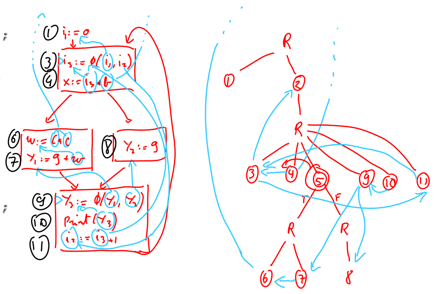


Another example

```
① i1 := 0;
  while ②... do
  ③ i3 := φ(i1, i2);
  ④ x := i3 * b;
    if ⑤... then
  ⑥   w := c * c;
  ⑦   y1 := 9 + w;
    else
  ⑧   y2 := 9;
    end
  ⑨ y3 := φ(y1, y2);
  ⑩ print(y3);
  ⑪ i2 := i3 + 1;
  end
```



Summary of Control Depence Graph

- More flexible way of representing control-depencies than CFG (less constraining)
- Makes code motion a local transformation
- However, much harder to convert back to an executable form

Course summary so far

- Dataflow analysis
  - flow functions, lattice theoretic framework, optimistic iterative analysis, precision, MOP
- Advanced Program Representations
  - SSA, CDG, PDG
- Along the way, several analyses and opts
  - reaching defns, const prop & folding, available exprs & CSE, liveness & DAE, loop invariant code motion
- Pointer analysis
  - Andersen, Steensguaard, and long the way: flow-insensitive analysis
- Next: dealing with procedures