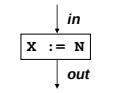
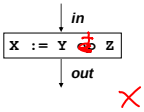
Another example: constant prop

- Set D = $2^{\{ x \to N \mid x \in Vars \wedge N \in Z \}}$



$F_{X := N}(in) = in - \{ X \to * \} \cup \{ X \to N \}$

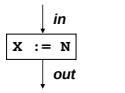


$F_{X := Y \text{ op } Z}(in) = in - \{ X \to * \} \cup$
$\{ X \to N \mid (Y \to N_1) \in in \wedge$
$(Z \to N_2) \in in \wedge$
$N = N_1 \text{ op } N_2 \}$

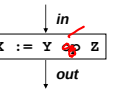
1

Another example: constant prop

- Set D = $2^{\{ x \to N \mid x \in Vars \wedge N \in Z \}}$



$F_{X := N}(in) = in - \{ X \to * \} \cup \{ X \to N \}$

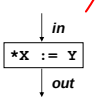


$F_{X := Y \text{ op } Z}(in) = in - \{ X \to * \} \cup$
$\{ X \to N \mid (Y \to N_1) \in in \wedge$
$(Z \to N_2) \in in \wedge$
$N = N_1 \text{ op } N_2 \}$

2

Another example: constant prop

$F_{X := *Y}(in) = in - \{ X \to * \} \cup$
$\{ X \to N \mid \forall V \in MPT(Y) : (V \to N) \in in \}$



$F_{*X := Y}(in) = in - \{ V \to * \mid V \in MPT(X) \}$

3

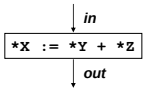
Another example: constant prop

$F_{X := *Y}(in) = in - \{ X \to * \}$
$\cup \{ X \to N \mid \forall Z \in \text{may-point-to}(Y) .$
$(Z \to N) \in in \}$

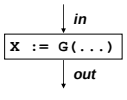
$F_{*X := Y}(in) = in - \{ Z \to * \mid Z \in \text{may-point}(X) \}$
$\cup \{ Z \to N \mid Z \in \text{must-point-to}(X) \wedge$
$Y \to N \in in \}$
$\cup \{ Z \to N \mid (Y \to N) \in in \wedge$
$(Z \to N) \in in \}$

4

Another example: constant prop



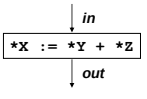
$F_{*X := *Y + *Z}(in) =$



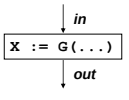
$F_{X := G(...)}(in) =$

5

Another example: constant prop



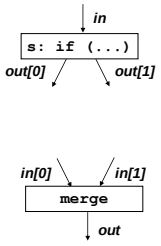
$F_{*X := *Y + *Z}(in) = F_{a := *Y; b := *Z; c := a + b; *X := c}(in)$



$F_{X := G(...)}(in) = \emptyset$

6

Another example: constant prop



7

Lattice

- (D, ⊑, ⊥, ⊤, ⊔, ⊓) =

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Lattice

- (D, ⊑, ⊥, ⊤, ⊔, ⊓) =
  $(2^A, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \rightarrow N \mid x \in Vars \wedge N \in Z \}$

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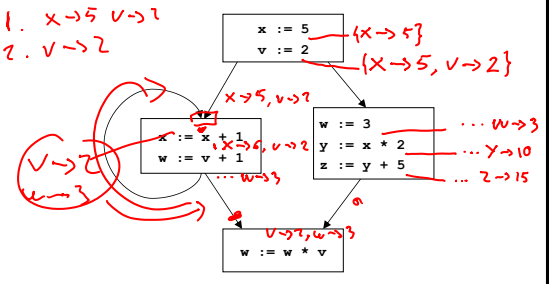
Example



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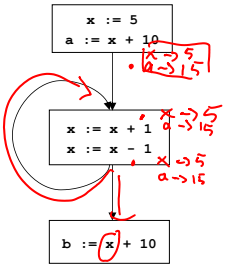
Another Example



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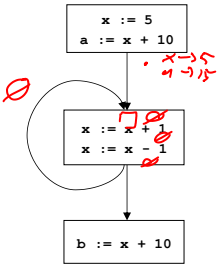
Another Example starting at top



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Back to lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^A, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \to N \mid x \in Vars \wedge N \in Z \}$
- What's the problem with this lattice?

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Back to lattice

- $(D, \sqsubseteq, \bot, \top, \sqcup, \sqcap) =$
  $(2^A, \supseteq, A, \emptyset, \cap, \cup)$
  where $A = \{ x \to N \mid x \in Vars \wedge N \in Z \}$
- What's the problem with this lattice?
- Lattice is infinitely high, which means we can't guarantee termination

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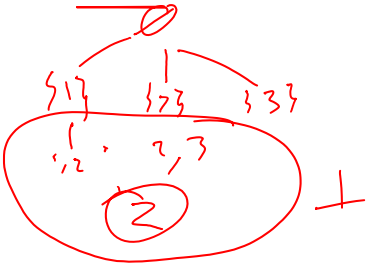
Better lattice

- Suppose we only had one variable



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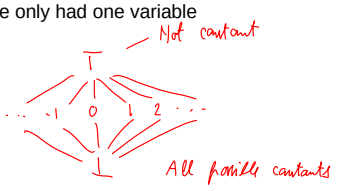
Better lattice

- Suppose we only had one variable



- $D = \{\bot, \top\} \cup Z$
- $\forall i \in Z . \bot \sqsubseteq i \wedge i \sqsubseteq \top$
- height = 3

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For all variables

- Two possibilities
- Option 1: Tuple of lattices
- Given lattices $(D_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1)$ ... $(D_n, \sqsubseteq_n, \bot_n, \top_n, \sqcup_n, \sqcap_n)$ create:

  tuple lattice D^n =

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For all variables

- Two possibilities
- Option 1: Tuple of lattices
- Given lattices $(D_1, \sqsubseteq_1, \bot_1, \top_1, \sqcup_1, \sqcap_1)$ ... $(D_n, \sqsubseteq_n, \bot_n, \top_n, \sqcup_n, \sqcap_n)$ create:

  tuple lattice $D^n = ((D_1 \times ... \times D_n), \sqsubseteq, \bot, \top, \sqcup, \sqcap)$ where
  $\bot = (\bot_1, ..., \bot_n)$
  $\top = (\top_1, ..., \top_n)$
  $(a_1, ..., a_n) \sqcup (b_1, ..., b_n) = (a_1 \sqcup_1 b_1, ..., a_n \sqcup_n b_n)$
  $(a_1, ..., a_n) \sqcap (b_1, ..., b_n) = (a_1 \sqcap_1 b_1, ..., a_n \sqcap_n b_n)$
  $height = height(D_1) + ... + height(D_n)$

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For all variables

- Option 2: Map from variables to single lattice
- Given lattice (D, $\sqsubseteq_1$, $\bot_1$, $\top_1$, $\sqcup_1$, $\sqcap_1$) and a set V, create:

map lattice V → D = (V → D, $\sqsubseteq$, $\bot$, $\top$, $\sqcup$, $\sqcap$)

$\bot = \lambda v \rightarrow \bot_1$

$\top = \lambda v \rightarrow \top_1$

$m_1 \sqcup m_2 = \lambda v \rightarrow m_1(v) \sqcup_1 m_2(v)$

$m_1 \sqsubseteq m_2 \iff \forall v.\ m_1(v) \sqsubseteq_1 m_2(v)$

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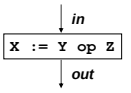
Back to example



$F_{X := Y \text{ op } Z}(in) = in[X \mapsto in(Y) \widehat{op} in(Z)]$

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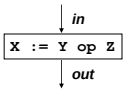
Back to example



$F_{X := Y \text{ op } Z}(in) = in [X \mapsto in(Y) \widehat{op} in(Z)]$

where $a \widehat{op} b$ =

$\widehat{op}$	$\bot$	d_1	$\top$
$\bot$	$\bot$	$\bot$	$\top$
d_2	$\bot$	$d_1 \widehat{op} d_2$	$\top$
$\top$	$\top$	$\top$	$\top$

+	1	2
3	4	5
4	5	6

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General approach to domain design

- Simple lattices:
  - boolean logic lattice
  - powerset lattice
  - incomparable set: set of incomparable values, plus top and bottom (eg const prop lattice)
  - two point lattice: just top and bottom
- Use combinators to create more complicated lattices
  - tuple lattice constructor
  - map lattice constructor

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May vs Must

{a→b, b→c}

- Has to do with definition of computed info
- Set of x → y must-point-to pairs
  - if we compute x → y, then, then during program execution, x must point to y
- Set of x→ y may-point-to pairs
  - if during program execution, it is possible for x to point to y, then we must compute x → y

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May vs must

	May	Must
most optimistic (bottom)	∅	FS
most conservative (top)	FS	∅
safe		
merge	∪	∩

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May vs must

	May	Must
most optimistic (bottom)	empty set	full set
most conservative (top)	full set	empty set
safe	overly big	overly small
merge	∪	∩

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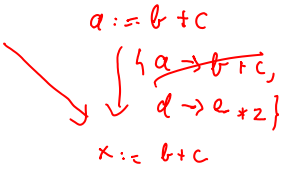
Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain:



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Common Sub-expression Elim

- Want to compute when an expression is available in a var
- Domain:

$S = \{ X \to E \mid X \in Var, E \in Exp \}$

$D = 2^S$
$\perp = S$
$T = \emptyset$
$\cup = \cap$

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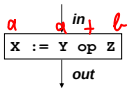
Flow functions



$F_{X := Y \text{ op } Z}(in) = in - \{ X \to * \} \cup \{ X \to Y \text{ op } Z \}$

$F_{X := Y}(in) =$

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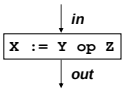
Flow functions



$F_{X := Y \text{ op } Z}(in) = in - \{ X \to * \} - \{ * \to \ldots X \ldots \} \cup \{ X \to Y \text{ op } Z \mid X \neq Y \wedge X \neq Z \}$

$F_{X := Y}(in) = in - \{ X \to * \} - \{ * \to \ldots X \ldots \} \cup \{ X \to E \mid Y \to E \in in \}$

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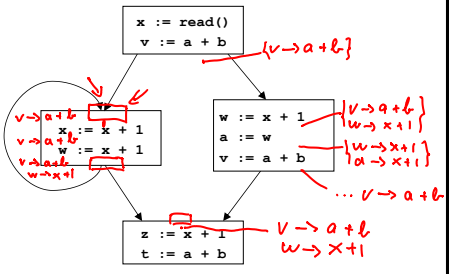
Example



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Direction of analysis

- Although constraints are not directional, flow functions are
- All flow functions we have seen so far are in the forward direction
- In some cases, the constraints are of the form
  in = F(out)
- These are called backward problems.
- Example: live variables
  – compute the set of variables that may be live

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Live Variables

- A variable is live at a program point if it will be used before being redefined
- A variable is dead at a program point if it is redefined before being used

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Example: live variables

- Set D =
- Lattice: (D, $\sqsubseteq$, $\bot$, $\top$, $\sqcup$, $\sqcap$) =

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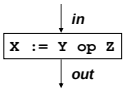
Example: live variables

- Set D = 2^{Vars}
- Lattice: (D, $\sqsubseteq$, $\bot$, $\top$, $\sqcup$, $\sqcap$) = (2^{Vars}, $\subseteq$, $\emptyset$,Vars, $\cup$, $\cap$)



$F_{X := Y \text{ op } Z}$(out) =

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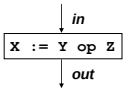
Example: live variables

- Set D = 2^{Vars}
- Lattice: (D, $\sqsubseteq$, $\bot$, $\top$, $\sqcup$, $\sqcap$) = (2^{Vars}, $\subseteq$, $\emptyset$,Vars, $\cup$, $\cap$)



$F_{X := Y \text{ op } Z}$(out) = out – { X } $\cup$ { Y, Z}

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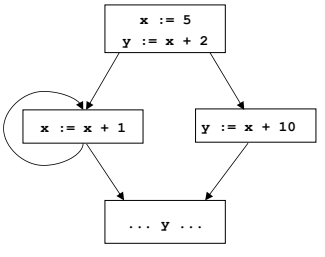
Example: live variables



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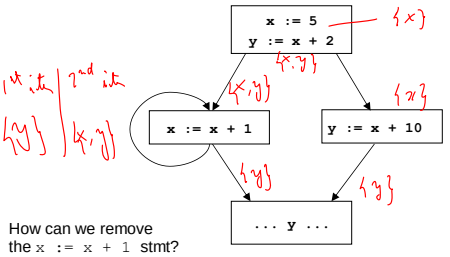
Example: live variables



How can we remove the `x := x + 1` stmt?

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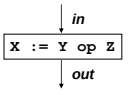
Revisiting assignment



$F_{X := Y \text{ op } Z}(out) = out - \{ X \} \cup \{ Y, Z \}$

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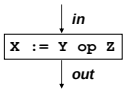
Revisiting assignment



$F_{X := Y \text{ op } Z}(out) = out - \{ X \} \cup \{ Y, Z \}$

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Theory of backward analyses

- Can formalize backward analyses in two ways
- Option 1: reverse flow graph, and then run forward problem
- Option 2: re-develop the theory, but in the backward direction

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Precision

- Going back to constant prop, in what cases would we lose precision?

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Precision

- Going back to constant prop, in what cases would we lose precision?

```
x := 5
if (<expr>) {
   x := 6
}
... x ...

where <expr> is
equiv to false
```

```
if (p) {
   x := 5;
} else
   x := 4;
}
...
if (p) {
   y := x + 1
} else {
   y := x + 2
}
... y ...
```

```
if (...) {
   x := -1;
} else
   x := 1;
}
y := x * x;
... y ...
```

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Precision

- The first problem: Unreachable code
  - solution: run unreachable code removal before
  - the unreachable code removal analysis will do its best, but may not remove all unreachable code
- The other two problems are path-sensitivity issues
  - Branch correlations: some paths are infeasible
  - Path merging: can lead to loss of precision

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MOP: meet over all paths

- Information computed at a given point is the meet of the information computed by each path to the program point

```
if (...) {
   x := -1;
} else
   x := 1;
}
y := x * x;
... y ...
```

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MOP

- For a path p, which is a sequence of statements $[s_1, ..., s_n]$, define: $F_p(in) = F_{s_n}(...F_{s_1}(in) ...)$
- In other words: $F_p =$ $\tilde{F}_{s_1} \circ \cdots \circ \tilde{F}_{s_n}$
- Given an edge e, let paths-to(e) be the (possibly infinite) set of paths that lead to e
- Given an edge e, MOP(e) = $\bigsqcup_{p \in paths\text{-}to(e)} F_p(\bot)$
- For us, should be called JOP (ie: join, not meet)

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MOP vs. dataflow

- MOP is the "best" possible answer, given a fixed set of flow functions
  - This means that MOP $\sqsubseteq$ dataflow at edge in the CFG
- In general, MOP is not computable (because there can be infinitely many paths)
  - vs dataflow which is generally computable (if flow fns are monotonic and height of lattice is finite)
- And we saw in our example, in general, MOP $\neq$ dataflow

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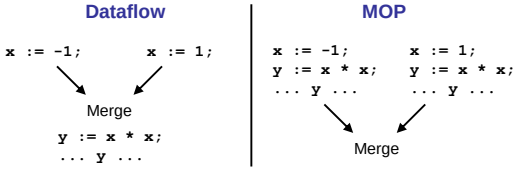
MOP vs. dataflow

- However, it would be great if by imposing some restrictions on the flow functions, we could guarantee that dataflow is the same as MOP. What would this restriction be?



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MOP vs. dataflow

- However, it would be great if by imposing some restrictions on the flow functions, we could guarantee that dataflow is the same as MOP. What would this restriction be?
- Distributive problems. A problem is distributive if:

  $\forall$ a, b . F(a $\sqcup$ b) = F(a) $\sqcup$ F(b)

- If flow function is distributive, then MOP = dataflow

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Summary of precision

- Dataflow is the basic algorithm
- To basic dataflow, we can add path-separation
  - Get MOP, which is same as dataflow for distributive problems
  - Variety of research efforts to get closer to MOP for non-distributive problems
- To basic dataflow, we can add path-pruning
  - Get branch correlation
- To basic dataflow, can add both:
  - meet over all feasible paths

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