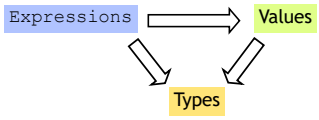
Programming Languages

Datatypes

UCSD

Review so far



Many kinds of expressions:

1. Simple
2. Variables
3. Functions

Review so far

- We've seen some base types and values:
  - Integers, Floats, Bool, String etc.
- Some ways to build up types:
  - Products (tuples), records, "lists"
  - Functions
- Design Principle: Orthogonality
  - Don't clutter core language with stuff
  - Few, powerful orthogonal building techniques
  - Put "derived" types, values, functions in libraries

Next: Building datatypes

Three key ways to build complex types/values

1. "Each-of" types
Value of T contains value of T1 **and** a value of T2

2. "One-of" types
Value of T contains value of T1 **or** a value of T2

3. "Recursive"
Value of T contains (sub)-value of **same type** T

Next: Building datatypes

Three key ways to build complex types/values

1. "Each-of" types **`(T1 * T2)`**
Value of T contains value of T1 **and** a value of T2

2. "One-of" types
Value of T contains value of T1 **or** a value of T2

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Suppose I wanted …

… a program that processed lists of attributes

- Name (string)
- Age (integer)
- …

Suppose I wanted …

… a program that processed lists of attributes

- Name (string)
- Age (integer)
- DOB (int-int-int)
- Address (string)
- Height (float)
- Alive (boolean)
- Phone (int-int)
- email (string)

Many kinds of attributes (too many to put in a record)

- can have multiple names, addresses, phones, emails etc.

Want to store them in a list. Can I ?

Constructing Datatypes

```
type t = C1 of t1 | C2 of t2 | … | Cn of tn
```

`t` is a new datatype.

A value of type `t` is either:

a value of type *t1* placed in a box labeled C1

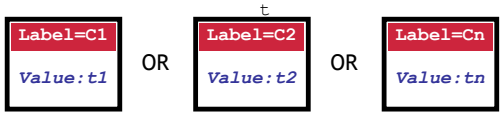
Or a value of type *t2* placed in a box labeled C2

Or …

Or a value of type *tn* placed in a box labeled Cn

Constructing Datatypes

```
type t = C1 of t1 | C2 of t2 | … | Cn of tn
```



All have the type **t**

Suppose I wanted …

Attributes:

- Name (string)
- Age (integer)
- DOB (int-int-int)
- Address (string)
- Height (real)
- Alive (boolean)
- Phone (int-int)
- email (string)

```
type attrib =
  Name of string
| Age of int
| DOB of int*int*int
| Address of string
| Height of float
| Alive of bool
| Phone of int*int
| Email of string;;
```

How to PUT values into box?

How to PUT values into box?

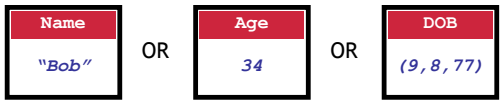
How to create values of type `attrib` ?

```
# let a1 = Name "Bob";;
val x : attrib = Name "Bob"
# let a2 = Height 5.83;;
val a2 : attrib = Height 5.83
# let year = 1977 ;;
val year : int = 1977
# let a3 = DOB (9,8,year) ;;
val a3 : attrib = DOB (9,8,1977)
# let a_l = [a1;a2;a3];;
val a3 : attrib list = …
```

```
type attrib =
  Name of string
| Age of int
| DOB of int*int*int
| Address of string
| Height of float
| Alive of bool
| Phone of int*int
| Email of string;;
```

Constructing Datatypes

```
type attrib
  = Name of string    | Age of int       | DOB of int*int*int
  | Address of string | Height of float | Alive of bool
  | Phone of int*int  | Email of string;;
```



Name "Bob" | Age 34 | DOB (9,8,77)

All have type **attrib**

One-of types

- We've defined a "one-of" type named `attrib`
- Elements are one of:
  - string,
  - int,
  - int*int*int,
  - float,
  - bool …
- Can create uniform `attrib` lists
- Say I want a function to print attribs…

```
datatype attrib =
  Name of string
| Age of int
| DOB of int*int*int
| Address of string
| Height of real
| Alive of bool
| Phone of int*int
| Email of string;
```

How to TEST & TAKE whats in box?

Is it a …
`string?`
or an
`int?`
or an
`int*int*int?`
or …

How to TEST & TAKE whats in box?

Look at TAG!

How to tell whats in the box ?

```
match e with
| Name s      -> printf "%s" s
| Age i       -> printf "%d" i
| DOB(d,m,y)  -> printf "%d/%d/%d" d m y
| Address s   -> printf "%s" s
| Height h    -> printf "%f" h
| Alive b     -> printf "%b" b s
| Phone(a,r)  -> printf "(%d)-%d" a r
```

Pattern-match expression: check if e is of the form …

- On match:
  - value in box bound to pattern variable
  - matching result expression is evaluated
- Simultaneously test and extract contents of box

How to tell whats in the box ?

```
type attrib =
  Name of string
| Age of int
| DOB of int*int*int
| Address of string
| Height of float
| Alive of bool
| Phone of int*int
```

```
match e with
| Name s     -> ...(*s: string *)
| Age i      -> ...(*i: int *)
| DOB(d,m,y)-> ...(*d: int,m: int,y: int*)
| Address a -> ...(*a: string*)
| Height h  -> ...(*h: int *)
| Alive b   -> ...(*b: bool*)
| Phone(a,r)-> ...(*a: int, r: int*)
```

Pattern-match expression: check if e is of the form …

- On match:
  - value in box bound to pattern variable
  - matching result expression is evaluated
- Simultaneously test and extract contents of box

How to tell whats in the box

```
# match (Name "Bob") with
   | Name s -> printf "Hello %s\n" s
   | Age i  -> printf "%d years old" i
  ;;
Hello Bob
- : unit = ()
```

None of the cases matched the tag (Name)
Causes nasty ***Run-Time Error***

How to TEST & TAKE whats in box?

BEWARE!!
Be sure to handle all TAGS!

Beware! Handle All TAGS!

```
# match (Name "Bob") with
   | Age i   -> Printf.printf "%d" I
   | Email s -> Printf.printf "%s" s
;;
Exception: Match Failure!!
```

None of the cases matched the tag (Name)
Causes nasty ***Run-Time Error***

Compiler to the Rescue!

```
# match (Name "Bob") with
   | Age i   -> Printf.printf "%d" I
   | Email s -> Printf.printf "%s" s
;;
Exception: Match Failure!!
```

None of the cases matched the tag (Name)
Causes nasty ***Run-Time Error***

Compiler To The Rescue!!

```
# let printAttrib a =  match a with
    | Name s -> Printf.printf "%s" s
    | Age i  -> Printf.printf "%d" I
    | DOB (d,m,y) -> Printf.printf "%d / %d / %d" d m y
    | Address addr -> Printf.printf "%s" addr
    | Height h -> Printf.printf "%f" h
    | Alive b -> Printf.printf "%b" b
    | Email e -> Printf.printf "%s" e
;;
Warning P: this pattern-matching is not exhaustive.Here is
an example of a value that is not matched:Phone (_, _)
```

Compile-time checks for:
missed cases: ML warns if you miss a case!

Compiler To The Rescue!!

```
# let printAttrib a = match a with
    | Name s -> Printf.printf "%s" s
    | Age i  -> Printf.printf "%d" I
    | DOB (d,m,y) -> Printf.printf "%d / %d / %d" d m y
  ...
    | Age i  -> Printf.printf "%d" i  ;;
Warning U: this match case is unused.
```

Compile-time checks for:
redundant cases: ML warns if a case never matches

Another Few Examples

```
# let printAttrib a = match a with
    | Name s -> Printf.printf "%s" s
    | Age i  -> Printf.printf "%d" I
    | DOB (d,m,y) -> Printf.printf "%d / %d / %d" d m y
  ...
    | Age i  -> Printf.printf "%d" i  ;;
Warning U: this match case is unused.
```

See code text file

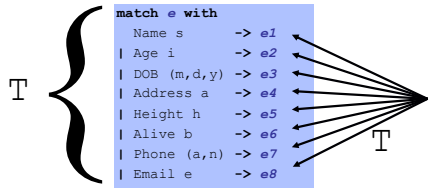
`match-with` is an Expression

```
match e with
  C1 x1 -> e1
| C2 x2 -> e2
| …
| Cn xn -> en
```

Type Rule

- *e1*, *e2*,…,*en* must have same type T
- Type of whole expression is T

`match-with` is an Expression



Type Rule

- *e1*, *e2*,…,*en* must have same type T
- Type of whole expression is T

Benefits of `match-with`

```
match e with
  C1 x1 -> e1
| C2 x2 -> e2
| …
| Cn xn -> en
```

```
type t =
  C1 of t1
| C2 of t2
| …
| Cn of tn
```

1. Simultaneous test-extract-bind
2. Compile-time checks for:
   missed cases: ML warns if you miss a `t` value
   redundant cases: ML warns if a case never matches

Next: Building datatypes

Three key ways to build complex types/values

1. "Each-of" types **`t1 * t2`**
Value of T contains value of T1 **and** a value of T2

2. "One-of" types **`type t = C1 of t1 | C2 of t2`**
Value of T contains value of T1 **or** a value of T2

3. "Recursive" type
Value of T contains (sub)-value of **same type** T

"Recursive" types

```
type nat = Zero | Succ of nat
```

"Recursive" types

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type nat = Zero | Succ of nat
```

Wait a minute! **Zero** of what ?!

"Recursive" types

```
type nat = Zero | Succ of nat
```

Wait a minute! **Zero** of what ?!

Relax.

Means "empty box with label **Zero**"

"Recursive" types

```
type nat = Zero | Succ of nat
```

What are values of **nat** ?

"Recursive" types

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One **nat** contains another!

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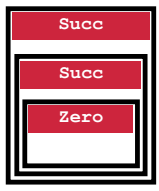
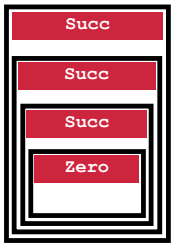


"Recursive" types

```
type nat = Zero | Succ of nat
```

What are values of **nat** ?
One **nat** contains another!

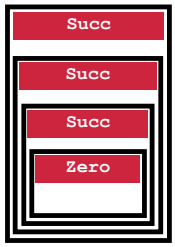


"Recursive" types

```
type nat = Zero | Succ of nat
```

What are values of **nat** ?
One **nat** contains another!

nat = recursive type



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Three key ways to build complex types/values

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Value of T contains value of T1 **and** a value of T2

2. "One-of" types **type t = C1 of t1 | C2 of t2**
Value of T contains value of T1 **or** a value of T2

3. "Recursive" type **type t = ...| C of (...*t)**
Value of T contains (sub)-value of **same type** T

Next: Lets get cosy with Recursion

Recursive Code Mirrors Recursive Data

Next: Lets get cosy with Recursion

Code Structure = Type Structure!!!

to_int : nat -> int

```
type nat =
| Zero
| Succ of nat
```

```
let rec to_int n =
```

to_int : nat -> int

```
type nat =
| Zero
| Succ of nat
```

Base pattern

Inductive pattern

```
let rec to_int n =
```

to_int : nat -> int

```
type nat =
| Zero
| Succ of nat
```

Base pattern

Inductive pattern

```
let rec to_int n = match n with
| Zero   -> 0
| Succ m -> 1 + to_int m
```

Base Expression

Inductive Expression

of_int : int -> nat

```
type nat =
| Zero
| Succ of nat
```

```
let rec of_int n =
```

of_int : int -> nat

```
type nat =
| Zero
| Succ of nat
```

Base pattern

Inductive pattern

```
let rec of_int n =
```

of_int : int -> nat

```
type nat =
| Zero
| Succ of nat
```

Base pattern

Inductive pattern

```
let rec of_int n =
  if n <= 0 then

  else
```

of_int : int -> nat

```
type nat =
| Zero
| Succ of nat
```

Base pattern

Inductive pattern

```
let rec of_int n =
  if n <= 0 then
    Zero
  else
    Succ (of_int (n-1))
```

Base Expression

Inductive Expression

plus : nat*nat -> nat

```
type nat =
| Zero
| Succ of nat

let rec plus n m =
```

plus : nat*nat -> nat

Base pattern
Inductive pattern

```
type nat =
| Zero
| Succ of nat

let rec plus n m =
```

plus : nat*nat -> nat

Base pattern
Inductive pattern

```
type nat =
| Zero
| Succ of nat

let rec plus n m =
match m with
| Zero   ->
| Succ m' ->
```

plus : nat*nat -> nat

Base pattern
Inductive pattern

```
type nat =
| Zero
| Succ of nat

let rec plus n m =
match m with
| Zero   -> n
| Succ m' -> Succ (plus n m')
```

Base Expression

Inductive Expression

times: nat*nat -> nat

```
type nat =
| Zero
| Succ of nat

let rec times n m =
```

times: nat*nat -> nat

Base pattern
Inductive pattern

```
type nat =
| Zero
| Succ of nat

let rec times n m =
```

times: nat*nat -> nat

```
type nat =
| Zero
| Succ of nat
```

Base pattern: `Zero`
Inductive pattern: `Succ of nat`

```
let rec times n m =
match m with
| Zero    ->
| Succ m' ->
```

Base pattern: `Zero`
Inductive pattern: `Succ m'`

times: nat*nat -> nat

```
type nat =
| Zero
| Succ of nat
```

Base pattern: `Zero`
Inductive pattern: `Succ of nat`

```
let rec times n m =
match m with
| Zero    -> Zero
| Succ m' -> plus n (times n m')
```

Base pattern: `Zero` → `Zero` — Base Expression
Inductive pattern: `Succ m'` → `plus n (times n m')` — Inductive Expression

Next: Lets get cosy with Recursion

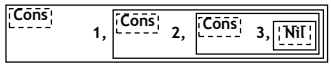
Recursive Code Mirrors Recursive Data

Lists are recursive types!

```
type int_list =
  Nil
| Cons of int * int_list
```

Think about this! What are values of `int_list` ?

`Cons(1,Cons(2,Cons(3,Nil)))` `Cons(2,Cons(3,Nil))` `Cons(3,Nil)` `Nil`



Lists aren't built-in !

```
datatype int_list =
  Nil
| Cons of int * int_list
```

Lists are a derived type: built using elegant core!

1. Each-of
2. One-of
3. Recursive

`::` is just a pretty way to say "Cons"
`[]` is just a pretty way to say "Nil"

Some functions on Lists : Length

```
let rec len l =
  match l with
  | Nil     -> 0
  | Cons(h,t) -> 1 + (len t)
```

Base pattern: `Nil` → `0` — Base Expression
Inductive pattern: `Cons(h,t)` → `1 + (len t)` — Inductive Expression

```
let rec len l =
  match l with
  | Nil -> 0
  | Cons(_,t) -> 1 + (len t)
```

No binding for head

```
let rec len l =
  match l with
  | Cons(_,t) -> 1 + (len t)
  | _ -> 0
```

Pattern-matching in order

Some functions on Lists : Append

```
let rec append (l1,l2) =
```

- Find the right induction strategy
  - Base case: pattern + expression
  - Induction case: pattern + expression

Well designed datatype gives strategy

Some functions on Lists : Max

```
let rec max xs =
```

- Find the right induction strategy
  - Base case: pattern + expression
  - Induction case: pattern + expression

Well designed datatype gives strategy

null, hd, tl are all functions …

Bad ML style: More than aesthetics !

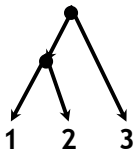
Pattern-matching better than test-extract:

- ML checks all cases covered
- ML checks no redundant cases
- …at compile-time:
  - fewer errors (crashes) during execution
  - get the bugs out ASAP!

Next: Lets get cosy with Recursion

Recursive Code Mirrors Recursive Data

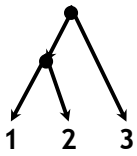
Representing Trees




```
type tree =
| Leaf of int
| Node of tree*tree
```

Leaf 1

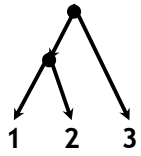
Representing Trees

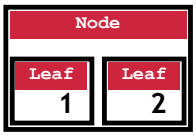



```
type tree =
| Leaf of int
| Node of tree*tree
```

Leaf 2

Representing Trees

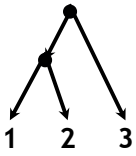





```
type tree =
| Leaf of int
| Node of tree*tree
```

```
Node(Leaf 1, Leaf 2)
```

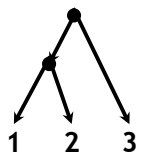
Representing Trees

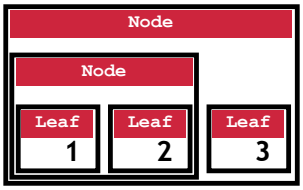



```
type tree =
| Leaf of int
| Node of tree*tree
```

```
Leaf 3
```

Representing Trees






```
type tree =
| Leaf of int
| Node of tree*tree
```

```
Node(Node(Leaf 1, Leaf 2), Leaf 3)
```

Next: Lets get cosy with Recursion

Recursive Code Mirrors Recursive Data

sum_leaf: tree -> int

"Sum up the leaf values". E.g.

```
# let t0 = Node(Node(Leaf 1, Leaf 2), Leaf 3);;
- : int = 6
```

sum_leaf: tree -> int

```
type tree =
| Leaf of int
| Node of tree*tree
```

```
let rec sum_leaf t =
```

sum_leaf: tree -> int

Base pattern
Inductive pattern

```
type tree =
| Leaf of int
| Node of tree*tree

let rec sum_leaf t =
```

sum_leaf: tree -> int

Base pattern
Inductive pattern

```
type tree =
| Leaf of int
| Node of tree*tree

let rec sum_leaf t =
match t with
| Leaf n      ->
| Node(t1,t2) ->
```

sum_leaf: tree -> int

Base pattern
Inductive pattern

```
type tree =
| Leaf of int
| Node of tree*tree

let rec sum_leaf t =
match t with
| Leaf n      -> n                  (* Base Expression *)
| Node(t1,t2) -> sum_leaf t1 + sum_leaf t2   (* Inductive Expression *)
```

Recursive Code Mirrors Recursive Data

Code almost writes itself!

Another Example: Calculator

Want an arithmetic calculator to evaluate expressions like:

- 4.0 + 2.9
- 3.78 - 5.92
- (4.0 + 2.9) * (3.78 -5.92)

Another Example: Calculator

Want an arithmetic calculator to evaluate expressions like:

- 4.0 + 2.9 ====> *6.9*
- 3.78 - 5.92 ====> *-2.14*
- (4.0 + 2.9) * (3.78 -5.92) ====> *-14.766*

Whats a ML **TYPE** for **REPRESENTING** expressions ?

Another Example: Calculator

Want an arithmetic calculator to evaluate expressions like:

- 4.0 + 2.9 ====> *6.9*
- 3.78 - 5.92 ====> *-2.14*
- (4.0 + 2.9) * (3.78 -5.92) ====> *-14.766*

Whats a ML **TYPE** for **REPRESENTING** expressions ?

```
type expr =
| Num of float
| Add of expr*expr
| Sub of expr*expr
| Mul of expr*expr
```

Another Example: Calculator

Want an arithmetic calculator to evaluate expressions like:

- 4.0 + 2.9 ====> *6.9*
- 3.78 - 5.92 ====> *-2.14*
- (4.0 + 2.9) * (3.78 -5.92) ====> *-14.766*

Whats a ML **FUNCTION** for **EVALUATING** expressions ?

```
type expr =
| Num of float
| Add of expr*expr
| Sub of expr*expr
| Mul of expr*expr
```

Another Example: Calculator

Want an arithmetic calculator to evaluate expressions like:

- 4.0 + 2.9 ====> *6.9*
- 3.78 - 5.92 ====> *-2.14*
- (4.0 + 2.9) * (3.78 -5.92) ====> *-14.766*

Whats a ML **FUNCTION** for **EVALUATING** expressions ?

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type expr =
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| Add of expr*expr
| Sub of expr*expr
| Mul of expr*expr
```

```
let rec eval e = match e with
|Num f      ->
|Add(e1,e2)->
|Sub(e1,e2)->
|Mul(e1,e2)->
```

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```
let rec eval e = match e with
|Num f      -> f
|Add(e1,e2)-> eval e1 +. eval e2
|Sub(e1,e2)-> eval e1 -. eval e2
|Mul(e1,e2)-> eval e1 *. eval e2
```